Restoration of Lost Corners
and
Subdivision of Sections

The Land Survey Section of the
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Preface

This document is intended to be a reference for the procedures used in the restoration of lost corners and subdivision of sections of public land surveys, specifically in the State of Washington. The Bureau of Land Management (BLM) Manual of Surveying Instructions, 2009, is the primary source. Some guidance has come from the BLM office in Portland, Oregon.

The system of public land surveys was devised for the purpose of settling and selling the lands of the public domain of the United States extending across the North American continent. Federal surveys did not establish every boundary and corner for each parcel of land granted by the government; the surveys established a network of surveyed lines and corners of sufficient detail to enable purchasers and owners of formerly public land to locate the corners and lines of their parcels. Subsequently, county and local surveyors were given the responsibility to establish many of the corners of those parcels.

Finding the original lines and corners is a responsibility of a land surveyor. If a previously established corner or line is not visible on the ground, the surveyor must try to find evidence of the corner or line. Only after exhaustive examination and research can a conclusion be reached that the corner or line is lost. If there is no record of the establishment of a corner not set by the Federal survey, the surveyor must nevertheless search for any evidence that the corner or line may have been already established. If any such evidence exists the rules and principles of acceptable corner evidence should be applied. A missing corner should be established using the principles for the restoration of lost corners or, if the corner has never been set previously, it is established using the principles for the establishment of original corners. The diligent retracement of previous surveys is necessary to avoid interference with the stability of land boundaries. Only when a retracement is unsuccessful or when no survey has previously been made can the procedures outlined here be applied.

A. C. Mulford, in his 1912 book, Boundaries and Landmarks, points out the importance of retracing the original survey. His observations, though directed to the surveyor of lands described by metes and bounds, apply equally to resurveys in the public land survey system. He instructs that

The training of the surveyor consists essentially in practice in turning angles, measuring lines and getting over obstructions, to which are added rather meager suggestions on the subjection of the compass and the re-running of old surveys. He is considered preeminently a measurer of land. This is very true, and in certain localities and under certain conditions this may compose almost the entire work of the surveyor. But in the vast majority of cases the actual measuring of land forms the smaller portion of his duties. His hardest work is often, to use a colloquial phrase, to “find the land” to be surveyed.

The United States Bureau of Land Management (BLM) and its predecessor, the General Land Office (GLO), have published manuals, pamphlets and circulars that cover the topics discussed here. The surveyor may use this document for guidance but should look for authoritative instructions in the relevant GLO and BLM documents and opinions, particularly the BLM Manual of Surveying Instructions and the BLM circular, Restoration of Lost or Obliterated Corners & Subdivision of Sections.
A Few Pertinent Land Survey Laws

Early Federal Law Summary

The United States, shortly after independence, began disposing areas of public land. The Land Ordinance of 1785, An Ordinance for Ascertaining the Mode of Disposing of Lands in the Western Territory, passed May 20, 1785, was the first of several legislative acts that created the Public Land System and the Rectangular System of Surveying. It established the basis of bearings for public land surveys to be “true,” which can mean either astronomic or geodetic bearings, depending on how the relationship of bearings to the earth’s rotation is determined.

The Act of May 18, 1796, created the office of Surveyor General and established the method by which townships are subdivided. The act delegated the authority to write surveying rules and regulations to the Surveyor General. Rufus Putnam was appointed as the first Surveyor General on October 1, 1796, and served until 1803 when Thomas Jefferson replaced him with Jared Mansfield, a technically oriented man who had been a professor of mathematics at West Point military academy. Albert Gallatin, a Swiss emigrant, was Jefferson’s Secretary of the Treasury and the supervisor of the Surveyor General. Jefferson, Gallatin, and Mansfield were all technically and scientifically oriented, and they attempted to insure that the surveys of the public lands would be systematic and rigorously executed.

The Act of February 11, 1805, crystallized the principles of the Public Land Survey System. The act is codified in Chapter 18 of Title 43 of the United States Code and is still law. The Act settled several basic principles of the subdivision of townships and sections.

1. Corners and lines established by United States survey are to be held as correct and proper corners and lines. No subsequent survey can correct the corner position to place it more in conformity with the plan of survey.
2. After an original survey, subdivisions are to be made by connecting opposite corners with straight lines. The method of quartering surveyed rectangular parcels such as sections is to connect the midpoint corners on opposite sides of the figure.
3. Where no such opposite corners exist, boundaries shall be made by running from established corners in a cardinal direction. Fractional sections and quarter sections, that do not have an opposite side midpoint, are to be subdivided by running cardinal.
4. The corners of half and quarter sections, not marked by the original survey, shall be placed as nearly as possible equidistant from the two corners which stand on the same line. Quarter-section corners and other corners such as sixteenth-section corners, which were not established by the federal survey, are to be established at midpoint.
5. The acreage returned by United States survey shall be considered as exact. Acreages shown on the plat of survey are to be used for land dispositions and apportionment of unsurveyed corners.

The words of the Act of February 11, 1805, soon required interpretation. On April 3, 1805, Surveyor General Jared Mansfield wrote to the Secretary of the Treasury, Albert Gallatin, and

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1 Much of this material is taken from A History of the Rectangular Survey System by C. Albert White, published by the Bureau of Land Management.
discussed the Act. His primary concern was the subdivision of fractional townships. The second clause of section 2 of the act said that fractional townships were to be subdivided by running due north and south or east and west from the corners to an intersection with the boundary, which made the township fractional. Mansfield advocated that this procedure wasn't proper unless the section boundaries were actually on a true cardinal bearing and that the subdvisional lines would have to be run parallel to the established section boundaries or mean courses would have to be adopted. This method of subdivision was adopted by Mansfield as following the intent of the law and is still the accepted practice.

A few weeks earlier, on March 13, 1805, Gallatin had written to Isaac Briggs, Surveyor of the Lands South of Tennessee, concerning the intent of the Act of February 11. His words make it clear that surveys were to be done in a practical manner and that the corners should remain fixed. Certainty of boundaries and corners was of paramount importance, with accuracy of surveys being secondary.

You will also perceive from the enclosed act that the principal object which Congress has in view is that the corners and boundaries of the sections & subdivision of sections should be definitively fixed; and that ascertainment of the precise contents of each is not considered as equally important. Indeed it is not so material either for the United States or for the individuals, that purchasers should actually hold a few acres more or less than their surveys may call for, as it is that they should know with precision, and so as to avoid any litigation, what are the certain boundaries of their tract.

The Acts of April 24, 1820, and April 5, 1832, added more provisions for the sale of public lands and the method of subdividing the land.

The Act of September 27, 1850, created the office of Surveyor General of the Public Lands in Oregon and provided for the survey and the making of Donations to the settlers of those Public Lands. The act is called the Donation Land Act. John B. Preston was appointed the first Surveyor General of Oregon. The surveys were to extend over the area of the current States of Oregon and Washington, and the Donations, because they required surveys of land claims that predated the surveys of the townships, were to result in thousands of special surveys that did not conform to the rectangular system, mostly in the Willamette Valley but also in the early settled areas of Washington such as Vancouver, Olympia and Port Townsend.

The Act of July 9, 1870, codified in Title 43, Chapter 18, Paragraph 766, United States Code, for the first time officially allowed county and local surveyors to subdivide surveyed public land. In practice, they had been subdividing sections since much earlier.

“and all subdividing of surveyed lands into lots less than one hundred and sixty acres may be done by county and local surveyors at the expense of claimants”
United States Code

The Public Land Survey System (PLSS) laws were incorporated into the United States Code as follows.

United States Code Title 43 - Public Lands - Chapter 18 - Survey of Public Lands
§ 751. Rules of survey
The public lands shall be divided by north and south lines run according to the true meridian, and by others crossing them at right angles, so as to form townships of six miles square, unless where the line of an Indian reservation, or of tracts of land surveyed or patented prior to May 18, 1796, or the course of navigable rivers, may render this impracticable; and in that case this rule must be departed from no further than such particular circumstances require.

Second. The corners of the townships must be marked with progressive numbers from the beginning; each distance of a mile between such corners must be also distinctly marked with marks different from those of the corners.

Third. The township shall be subdivided into sections, containing, as nearly as may be, six hundred and forty acres each, by running parallel lines through the same from east to west and from south to north at the distance of one mile from each other, and marking corners at the distance of each half mile. The sections shall be numbered, respectively, beginning with the number one in the northeast section and proceeding west and east alternately through the township with progressive numbers, until the thirty-six be completed.

Fourth. The deputy surveyors, respectively, shall cause to be marked on a tree near each corner established in the manner described, and within the section, the number of such section, and over it the number of the township within which such section may be; and the deputy surveyors shall carefully note, in their respective field books, the names of the corner trees marked and the numbers so made.

Fifth. Where the exterior lines of the townships which may be subdivided into sections or half-sections exceed, or do not extend six miles, the excess or deficiency shall be specially noted, and added to or deducted from the western and northern ranges of sections or half-sections in such township, according as the error may be in running the lines from east to west, or from north to south; the sections and half-sections bounded on the northern and western lines of such townships shall be sold as containing only the quantity expressed in the returns and plats respectively, and all others as containing the complete legal quantity.

Sixth. All lines shall be plainly marked upon trees, and measured with chains, containing two perches of sixteen and one-half feet each, subdivided into twenty-five equal links; and the chain shall be adjusted to a standard to be kept for that purpose.

Seventh. Every surveyor shall note in his field book the true situations of all mines, salt licks, salt springs, and mill-seats which come to his knowledge; all watercourses over which the line he runs may pass; and also the quality of the lands.

Eighth. These field books shall be returned to the Secretary of the Interior or such officer as he may designate, who shall cause therefrom a description of the whole lands surveyed to be made out and transmitted to the officers who may superintend the sales. He shall also cause a fair plat to be made of the townships and fractional parts of townships contained in the lands, describing the subdivisions thereof, and the marks of the corners. This plat shall be recorded in books to be kept for that purpose; and a copy thereof shall be kept open at the office of the Secretary of the Interior or of such agency as he may designate for public information, and other copies shall be sent to the places of the sale, and to the Bureau of Land Management.
§ 752 Boundaries and contents of public lands; how ascertained

First. All the corners marked in the surveys, returned by the Secretary of the Interior or such agency as he may designate, shall be established as the proper corners of sections, or subdivisions of sections, which they were intended to designate; and the corners of half- and quarter-sections, not marked on the surveys, shall be placed as nearly as possible equidistant from two corners which stand on the same line.

Second. The boundary lines, actually run and marked in the surveys returned by the Secretary of the Interior or such agency as he may designate, shall be established as the proper boundary lines of the sections, or subdivisions, for which they were intended, and the length of such lines as returned, shall be held and considered as the true length thereof. And the boundary lines which have not been actually run and marked shall be ascertained, by running straight lines from the established corners to the opposite corresponding corners; but in those portions of the fractional townships where no such opposite corresponding corners have been or can be fixed, the boundary lines shall be ascertained by running from the established corners due north and south or east and west lines, as the case may be, to the watercourse, Indian boundary line, or other external boundary of such fractional township.

Third. Each section or subdivision of section, the contents whereof have been returned by the Secretary of the Interior or such agency as he may designate, shall be held and considered as containing the exact quantity expressed in such return; and the half sections and quarter sections, the contents whereof shall not have been thus returned, shall be held and considered as containing the one-half or the one-fourth part, respectively, of the returned contents of the section of which they may make part.

Derived from Act Feb. 11, 1805

§ 753. Lines of division of half quarter sections; how run

In every case of the division of a quarter section the line for the division thereof shall run north and south, and the corners and contents of half quarter sections which may thereafter be sold, shall be ascertained in the manner and on the principles directed and prescribed by section 752 of this title, and fractional sections containing one hundred and sixty acres or upwards shall in like manner as nearly as practicable be subdivided into half quarter sections under such rules and regulations as may be prescribed by the Secretary of the Interior, and in every case of a division of a half quarter section, the line for the division thereof shall run east and west, and the corners and contents of quarter quarter sections, which may thereafter be sold, shall be ascertained as nearly as may be, in the manner, and on the principles, directed and prescribed by the section preceding; and fractional sections containing fewer or more than one hundred and sixty acres shall in like manner, as nearly as may be practicable, be subdivided into quarter quarter sections, under such rules and regulations as may be prescribed by the Secretary of the Interior.

Derived from Acts Apr. 24, 1820 and Apr. 5, 1832

§ 766. Geological surveys, extension of public surveys, expenses of subdividing and all subdividing of surveyed lands into lots less than one hundred and sixty acres may be done by county and local surveyors at the expense of claimants;

Derived from Act of July 9, 1870
Once public land is patented and passes out of federal control, the further subdivision of that land becomes subject to state law. The Washington Administrative Code has established that the GLO and BLM methods are the legally prescribed methods for the State and additionally requires that surveys show the method of subdivision and the controlling corners.

WAC 332-130-030 Land subdivision and corner restoration standards--Recording.

(1) The reestablishment of lost GLO or BLM corners and the subdividing of sections shall be done according to applicable GLO or BLM plats and field notes and in compliance with the rules as set forth in the appropriate GLO or BLM Manual of Surveying Instructions, manual supplements and circulars. Federal or state court decisions that influence the interpretation of the rules should be considered. Methods used for such corner reestablishment or section subdivision shall be described on the survey map produced.

(2) All maps, plats, or plans showing a land boundary survey shall show all the corners found, established, reestablished and calculated, including corresponding directions and distances, which were used to survey and which will be necessary to resurvey the parcel shown. Additionally, all such maps, plats, or plans shall show sufficient section subdivision data, or other such controlling parcel data, necessary to support the position of any section subdivisional corner or controlling parcel corner used to reference the parcel surveyed. Where a portion or all of this information is already shown on a record filed or recorded in the county recording office of the county in which the parcel is located, reference may be made to that record in lieu of providing the required data.

The Revised Code of Washington, 18.43.020, first defines a land surveyor and then defines the practice of land surveying.

(6) "Land surveyor" means a professional land surveyor.

(7) "Professional land surveyor" means a person who, by reason of his or her special knowledge of the mathematical and physical sciences and principles and practices of land surveying, which is acquired by professional education and practical experience, is qualified to practice land surveying and as attested to by his or her legal registration as a professional land surveyor.

(9) "Practice of land surveying" means assuming responsible charge of the surveying of land for the establishment of corners, lines, boundaries, and monuments, the laying out and subdivision of land, the defining and locating of corners, lines, boundaries, and monuments of land after they have been established, the survey of land areas for the purpose of determining the topography thereof, the making of topographical delineations and the preparing of maps and accurate records thereof, when the proper performance of such services requires technical knowledge and skill.

Washington law gives persons with state registration as a professional land surveyor the authority to survey “land for the establishment of corners, lines, boundaries, and monuments.” Washington surveyors can establish original corners and monuments for those corners.
Washington law also gives registered surveyors the authority to resurvey previously established corners and lines with the words authorizing “the defining and locating of corners, lines, boundaries, and monuments of land after they have been established.”

In short, Washington law gives registered land surveyors the authority to conduct both original and retracement surveys.

**Circulars and Manuals**

GLO and BLM circulars and manuals are rules for federal surveys and have the effect of law in States, such as Washington, that adopt them into law. The first GLO corner restoration and section subdivision circular, published in 1883 summarized Congressional legislation in five points. The wording has changed somewhat, but the same five points appear in the BLM circular of 1974, which are quoted below.

*The general rules followed by the Bureau of Land Management, which are controlling upon the location of all public lands, are summarized in the following paragraphs:

  First: That the boundaries of the public lands, when approved and accepted, are unchangeable.

  Second: That the original township, section, and quarter-section corners must stand as the true corners which they were intended to represent whether in the place shown by the field notes or not.

  Third: That quarter-quarter-section corners not established in the original survey shall he placed on the line connecting the section and quarter-section corners, and midway between them, except on the last half mile of section lines closing on the north and west boundaries of the township, or on the lines between fractional or irregular sections.

  Fourth: That the center lines of a section are to be straight, running from the quarter section corner on one boundary to the corresponding corner on the opposite boundary.

  Fifth: That in a fractional section where no opposite corresponding quarter section corner has been or can be established, the center line must he run from the proper quarter-section corner as nearly in a cardinal direction to the meander line, reservation, or other boundary of such fractional section, as due parallelism with the section boundaries will permit.

  From the foregoing it will he evident that corners established in the public land surveys remain fixed in position and are unchangeable; and that lost or obliterated corners of those surveys must be restored to their original locations from the best available evidence of the official survey in which such corners were established.*
The Public Land Survey System
PLSS
The principles of the PLSS must be taken into account when restoring lost corners and subdividing sections. The laws in the previous section outline the principles. The map above shows the areas of the United States covered by the PLSS and shows the baselines and meridians that control the survey.

Figure 3-1 from the 2009 Manual shows the construction of townships starting at the Initial Point, which is the intersection of the baseline and principle meridian.

Figure 3-1. Survey of quadrangles, each embracing 16 townships bounded by standard lines, showing the coordinate system of numbering townships.
Figure 3-25 shows a township with the sections numbered and the order of running the sections lines shown.

Figure 3-25. Sequence of numbers on section lines showing normal order of subdivision.
Methods of Survey

From the methods of survey described in Chapter 2 of 2009 Manual it can be seen that PLSS surveys cannot be properly retraced and sections cannot be correctly subdivided using the methods of cartesian coordinate geometry. This is the result of two characteristics of the PLSS.

First, distances in the PLSS are mean ground horizontal measurements. See the 2009 Manual, section 2-2. Consequently, when reproducing distances from the Federal record care must be taken to produce the record horizontal ground distance, which is different from a grid distance or a sea level distance. Due to elevation differences, a ground distance in one area of the survey might be represented by a different grid distance than the same ground distance in another area of the survey.

Second, the meridian for each individual line in the PLSS is the true meridian as defined by the axis of the earth’s rotation. See Manual section 2-3. Figure 2-1 in the Manual illustrates the effect of the PLSS basis of bearings.

Figure 2-1. Lines on exaggerated converging meridians.

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2 2009 Manual section 2-2. The distance reported is the horizontal measurement at the mean ground elevation for the line above sea level. Historically, distances have been measured on the ground surface, either horizontally or on the slope with vertical angle reduction to horizontal equivalent.

3 2009 Manual section 2-3. The direction of each line of the public land surveys is determined with reference to the true meridian as defined by the axis of the earth’s rotation. The true meridian is a line along a meridian of longitude. Historically, determination of the true meridian has been based upon direct astronomic observation at the point of record and, thus, an astronomic meridian.
“Straight Line” versus “Line of Sight”

In figure 2-1 line 1 is what private and geodetic surveys call a straight line, but the PLSS calls a “line of sight.” In the PLSS line 1, a line of sight, is a line of constantly changing bearing because at each point along line 1 the line is at a different angle to the true meridian. Line 2 is a rhumb line, a line of constant bearing, which in the PLSS is called a “straight line.” Township and section lines are straight lines. Lines of sight are relatively rare in the PLSS. They can occur along some State boundaries. A line of sight would have a different bearing between each corner on the line. The bearing is recorded as the mean bearing of the line, which is the mean of the bearings at the endpoints of the line. Most lines in the PLSS are straight lines, as in line 2. See Manual section 2-13. In summary, the PLSS survey lines are rhumb lines, called straight lines in the PLSS and called curved or rhumb lines by private and geodetic surveyors.

Because the PLSS employs horizontal ground distances and straight lines interpreted as rhumb lines, a PLSS survey will not close mathematically using the methods of cartesian coordinate geometry.

The Washington Administrative Code requires the use of the methods of the BLM Manual in the subdivision of sections. A possible conclusion is that State licensed section subdivision surveys should have converging meridians and boundaries that are rhumb lines, with the consequence that State surveys would not close using ordinary surveying coordinate geometry. Most, if not all, Washington land surveyors ignore the difference and survey using lines that the PLSS calls lines of sight. One exception is the case where there is extensive loss of corners along a township line and the curvature of the rhumb line must be taken into account in locating lost corners in order to avoid large errors. A practical point of view might recommend that, in the cases where the error produced by using lines of sight are small, one should employ the methods of plane cartesian coordinate geometry for calculating corner restorations and section subdivisions. In those cases where plane geometry cannot suffice, the BLM computer program, Cadastral Measurement Management, CMM, is recommended.

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4 The technical terms “straight line” and “line of sight” will be employed several times in this document and will usually be quoted or italicized as a reminder that the meanings are different from what might be assumed.
5 From the Glossary of BLM Surveying and Mapping Terms: Rhumb Line — A line on the surface of the earth making the same angle with all meridians; a loxodrome or loxodromic curve spiraling toward the poles in a constant true direction. Parallels and meridians, which also maintain constant true directions, may be considered special cases of the rhumb line.
6 2009 Manual section 2-13. By statute, in the PLSS datum, the term “straight line” is used when describing a line of constant bearing. The term has a different meaning from that used in geodesy, where a straight line corresponds approximately to a line of sight or a geodesic. ... Most lines in the PLSS are intended to be surveyed as lines of constant bearing. This is a direct result of the requirement that the lines be run 'according to the true meridian,' thereby crossing each meridian at the same angle. Other terms used for such lines are rhumb lines, small circles, or loxodromes.
The Local Surveyor

The BLM refers to non-Federal surveys and surveyors as “local” surveys and surveyors. The Act of July 9, 1870, gave local surveyors authority to subdivide sections. The words of the 2009 Manual describe the relationship between federal and local surveys.

The Manual recognizes the authority of the local surveyor and describes when federal surveyors should accept the corners marked by local surveys.

3-94. A local survey made for the purpose of marking on the ground a protracted line, platted but not run by the Government, where executed within the allowable limit of error for an original survey of that date, and relied upon by an owner under title passed by the United States in the placing of improvement upon the patented land, will not be disturbed, but it will be adopted by the Government as a boundary for closure of the survey of the adjoining public land.

Several sections in the Manual describe the functions and responsibilities of the local surveyor.

3-132. The work of the local surveyor usually includes the subdivision of the section into legal subdivisions shown upon the approved plat. In this capacity, the local surveyor is performing a function contemplated by law. He or she cannot properly serve the client or the public unless familiar with the legal requirements concerning the subdivision of sections.

3-133. In the event that the original monuments have become obliterated or lost, the local surveyor cannot hope to effectively recover the corner positions without a full understanding of the record concerning their original establishment and other evidence of establishment, subsequent recovery, or reestablishment. Nor can the local surveyor hope to legally restore or weigh evidence of subsequent corner location, use, occupancy, until he or she has mastered not only the principles observed in the execution of the original survey, and later local practices, but also the principles upon which the courts and authorized administrative officials having jurisdiction over such matters have based their rulings.

5-5. The function of county or other local surveyors begins when the surveyors undertake the identification of lands that have passed from the Government into private ownership, based upon the description derived from the original survey.

6-6. The function of the local surveyor begins when employed as an expert to identify lands that have passed into private ownership. The testimony or records of local surveyors who have identified the original monument prior to its destruction, or who have reasonably applied the good faith location rule, or who have marked the corners of legal subdivisions according to the prevailing law using the accuracy standards for the time and locale, is often considered reliable collateral evidence of the original surveyed and protracted lines and corners, particularly where those surveys are followed by use and occupancy by the landowners (section 3-132).

The Manual describes what circumstances would overturn a local survey. The local survey is to be accepted unless there is evidence of gross error or evidence of substantial non-conformance with the principles of corner recovery, restoration, and section subdivision.

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8 2009 Manual section 5-4. A local survey is any survey, retracement, or remonumentation of township, section, subdivision-of-section, or special survey lines that is not an official survey.
3-137. The law presupposes the fact taught by experience that measurements of lands cannot be repeated with absolute precision and that the work of no two surveyors will exactly agree. The governing law, 43 U.S.C. 752(2), states that “boundary lines which have not been actually run and marked shall be ascertained, by running straight lines from the established corners to the opposite corresponding corners.” The protracted position of the legal subdivision corner on the survey plat is merely the first step in fixing the position of the corner. The corner position is fixed by the running and marking of the lines.

A decision to set aside previously fixed local survey legal subdivision corners must be supported by evidence that goes beyond mere demonstration of technical error, reasonable discrepancies between former and new measurement, and less than strict adherence to restoration and subdivision rules. Were the Federal Government obliged to open the question as to the location of a particular tract or tracts over technical differences or reasonable discrepancies, controversies would constantly arise, and resurveys and readjudication would be interminable. The law gives these activities repose.

It is unlawful for the surveyor to impair bona fide rights as to location. Proof of impairment of bona fide rights as to location per 43 U.S.C. 772, when lines have been run and marked and corners marked and fixed by local survey, must be positive evidence of an intentional departure from the legal principles governing recovery of original corner location, reestablishment and establishment of corner location, or subdivision of section. Where the evidence of an extant subdivision-of-section survey indicates (1) a good faith attempt to relate it to the original controlling survey, (2) conformance as nearly as possible to legal subdivision principles, (3) reasonable accuracy standards for that time and place, (4) sufficiency for identification of the legal subdivisions, and (5) without fraud or gross error, the statutory intent of stability of boundaries and title to lands will have been met.

The Manual provides guidance for both federal and local surveyors.

6-4. On the other hand, it often falls to the county or other local surveyor to mark the corners of subdivisions or sections and the location of private property lines, and where a required corner is obliterated, the local surveyor may be called upon to recover the point. Thus it will be seen that local surveyors as well as cadastral surveyors of the BLM are constantly called upon to search for existing evidence of original monuments, and in this work the surveyors should be guided by the same general methods. The text that follows draws no distinction between these duties of the two classes of surveyors.
Restoration of Lost Corners

The position for a corner of the public land surveys may be recovered by reference to the record bearing trees or bearing objects.

From the BLM 1974 circular, Restoration of Lost or Obliterated Corners & Subdivision of Sections.
Principles for the Restoration of Lost Federal Survey Corners

The Act of February 11, 1805, and Title 43, Chapter 18, of the United States Public Code, provide that the corners of federal surveys are not subject to change. The corners are located wherever the federal surveyor put the monument, so long as the survey was approved, and they cannot be moved to a location more in harmony with the intent of the federal survey or to where the plat of the federal survey shows the corner to be by distance and direction from other corners.

If the original corner position is lost, then the surveyor must ensure that the corner is located where it will be accepted by the courts, which, in the State of Washington, means using the methods provided by BLM instructions. Because the courts will recognize the original corner position as correct, the surveyor must be certain that the corner is actually lost before resorting to the techniques for the restoration of lost corners. In addition to following BLM procedures the surveyor may be required to adhere to relevant state laws and court decisions.

An important theme in the BLM rules for restoring lost corners is the attempt to put the corner as close as possible to its original position as defined by the plat and field notes. The surveyor does not try to put the corner monument where it ought to have been placed. He tries to put it where it was actually established. Although the last resort, reliance on distances to other federal corners, i.e., proportioning, will not normally result in the restoration of a corner to its original location, it will result in a position that is most closely related to the survey record.

Types of Corners

A corner and a monument are different. The 2009 Manual, section 6-8, states that

The terms “corner” and “monument” are not interchangeable. A “corner” is a point determined by the surveying process. A “monument” is the object or the physical structure that marks the corner.

An Existent Corner can be monumented at its original location. A corner can be judged existent not only from its original monument or accessories, but also through other substantial evidence such as an “acceptable supplemental survey record” that locates it. An acceptably accurate survey with connections to the corner monument before it was destroyed can make the corner existent.

6-11. An existent corner is one whose original position can be identified by substantial evidence of the monument or its accessories, by reference to the description in the field notes, or located by an acceptable supplemental survey record, some physical evidence, or reliable testimony. A corner is existent (or found) if such conclusion is supported by substantial evidence. ... Substantial evidence is more than a scintilla of evidence but less than a preponderance of the evidence.

An Obliterated Corner is a type of Existent Corner.

6-17. An obliterated corner is an existent corner where at the corner’s original position, there are no remaining traces of the monument or its accessories but whose position has been perpetuated, or the point for which may be recovered, by substantial evidence from the acts or
reliable testimony of the interested landowners, competent surveyors, or other qualified local authorities, or witnesses, or by some acceptable record evidence.

A Lost Corner cannot generally be monumented at its original location. A lost corner is one whose original position cannot be determined by substantial evidence, either from traces of the original marks or from acceptable evidence or reliable testimony that bears upon the original position, and whose location can be restored only by reference to one or more interdependent corners. Thus, if substantial evidence of the position of the original corner exists, it is an existent or obliterated corner.

Evidence of Existent and Obliterated Corners

Before declaring a corner to be lost several avenues of evidence should be explored. The best evidence is the original monument or its accessories. Manual section 6-19 lists other features that can be evidence of an obliterated corner: line trees, witness points, connections to natural objects or improvements, mean positions of blazed lines, and mean position of a fence line or other occupancy line.

Manual section 5-9 stresses the importance of roads, fences, and other improvements.

In conducting a resurvey, care must be exercised to avoid disturbing satisfactory local conditions such as roads, fences, or other improvements marking subdivision-of-section lines and that may correctly define the extent of established bona fide property rights.

Section 2-34 notes that coordinates can be used for restoration of obliterated corners.

Repeatable coordinates may provide collateral evidence of a corner position, may constitute the best available evidence of a corner position, and, in some cases, may constitute substantial evidence of the position of an obliterated corner.

Section 6-26 states that topographic calls are best employed as supplementary evidence, not the primary or only evidence of a corner location.

The determination of the original corner point from even fragmentary evidence of the original accessories, generally substantiated by the original topographic calls, is much stronger than determination from topographic calls alone. In questionable cases it is better practice, in the absence of other collateral evidence, to turn to the suitable means of proportionate measurement.

Local corners, corners set by local surveys, can be the best evidence of the original corner location.

6-45. Once a local point of control is accepted in an official survey it has all the authority and significance of an original corner. … Chief among this class of evidence forming the basis of recognized land boundaries are: recorded monuments established by local surveyors and duly agreed upon by interested property owners; the position of boundary fences determined in

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9 The accuracy of the survey record used to proportion a lost corner governs the ability to restore it to its original location.
the same manner; and the lines of public roads, drainage or irrigation ditches, and timber cutting lines; when intended to be located with reference to the original subdivision lines. ... If a point qualifies for acceptance, having satisfied the requirement for substantial agreement with evidence of the original survey, the presumption is strong that its position bears satisfactory relation to the original survey and the burden of proof to the contrary must be borne by the party claiming differently. Points that so qualify must be accepted as the best available evidence of the true position of the original survey.

6-48. The age, position, and degree to which a local corner has been relied on by all affected landowners may lead to its adoption as the best remaining evidence of the position of the original corner. When a local reestablishment of a lost corner or a local establishment of a legal subdivision corner has been made by proper methods without gross error, it will ordinarily be acceptable. Monuments of unknown origin must be judged on their own merits, but these monuments should never be rejected out of hand without careful study.

In the case of two monuments claiming to represent one corner, the first monument in time gets preferential treatment.

5-7. Often the surveyor is faced with the case of two monuments, each purporting to mark the same corner, each linked to the original survey, and each identified by substantial evidence of the original corner position. Generally, within the PLSS, the second position in time will have the burden to prove, by a preponderance of the evidence, that the first position is not an accurate retracement and reestablishment of the lines of the original survey.

Steps in the Restoration of a Lost Corner.

First: Obtain all the GLO and/or BLM records that pertain to the corner, both the plats and the field notes. In the state of Washington some of those records were destroyed. At 2 AM Wednesday, September 12, 1883, the Surveyor General’s Office in Olympia burst into flame and destroyed the paper records. The cause of the fire was a discarded smoldering cigar butt. The original plats and field notes were destroyed. There was a duplicate record in Washington, DC, which was transcribed and conveyed to Olympia. The BLM calls this record the “transcribed duplicate” and the DNR Public Land Survey Office (PLSO) may call it the “triplicate.” Copies of the transcribed duplicate and the Washington, DC, duplicate can be obtained from the PLSO. The Manual recommends caution in the use of the transcribed duplicate.

7-6. Cases arise where the original survey record has been destroyed and the copies immediately available to the surveyor are transcribed copies of the duplicate record. If the field conditions do not match the transcribed duplicate record, the duplicate record will be compared to the transcribed copies to assure accuracy.

Second: Determine which are the existing and restored corners or monuments that control the lost corner and retrace in the field the lines between the lost corner vicinity and the controlling corners or monuments for the restoration.

Third: Remove manifest blunders from the measurement record to be used in the restoration.

7-5. All discrepancies in measurement will be verified with the object of placing each difference where it properly belongs. Manifest blunders in measurement are removed from the general average differences and placed where the blunder was made. In cases where the proportioned
position cannot be made to harmonize with all the calls of the original field notes, due to errors in description or to discrepancies in measurement made apparent by the retracement, it should be ascertained which of the calls are entitled to greater weight and which calls should be subordinate. The accumulated surplus or deficiency that remains is to be uniformly distributed by proportionate measurement.

Fourth: Apply the appropriate proportionate measurement.

7-7. A proportionate measurement is one that gives equal relative weight to all parts of the line based upon the process conforming to the method followed in the original survey. The excess or deficiency between two existent corners is so distributed that the amount given to each interval bears the same proportion to the whole difference as the record length bears to the whole record distance. … Relative to proportionate measurement in order to harmonize the restorative process with the methods of the original survey, the principle of the precedence of one line over another of less original importance is recognized, thus limiting the control.
Summary of Proportioning Applications

When to Double Proportion (Manual 7-10, 7-12)
- Township corner common to four townships
- Section corners of four sections interior to a township, where all the lines have been run

When to use Three Point Control (Manual 7-13)
- Township corner or section corner, interior to a township, where the line has not been established in one direction

When to use Two Point Control (Manual 7-14)
- Township or section corner, interior to a township, where intersecting lines have been established in two directions only

When to Single Proportion (Manual 7-16 to 7-49)
- In general, all corners on a line with measurements to adjoining corners in both directions
- Corners on a standard parallel
- Corners on base lines
- Corners on township boundaries established as a straight line, except township corners
- Corners on base lines, standard parallels, and correction lines
- Quarter-section corners interior to a township, unless they were measured from one direction only
- Meander corners on a line measured across a body of water
- Closing corners

When to use Irregular Boundary Adjustment (Manual 7-51, 7-52)
- Township boundaries not established as a straight line
- Township or section lines shown to be irregular, or not straight, in a retracement

A Special Method for Meander Lines (Manual 7-53)

When to use the Grant Boundary Method (Manual 7-54)
- Reservation boundaries

No hard and fast rule (Manual 7-55)
- Donation land claims
- Homestead entry surveys
- Mineral surveys
- Tract corners
- Townsites

When to use Single Point Control (Manual 7-56)
- Any corner with a measurement to another corner in one direction only
- Examples are terminal meander corners and some corners where the original survey terminated without completing the township or township subdivision survey
Double Proportion

General Description:

7-8. The term "double proportionate measurement" is applied to a new measurement made between four known corners, two each on intersecting meridional and latitudinal lines, for the purpose of relating the cardinal equivalents intersection to both.

In effect, by double proportionate measurement the record directions are disregarded ... Corners to the north and south control any latitudinal position. Corners to the east and west control the position in longitude. ... Each identified corner is given a controlling weight inversely proportional to its distance from the lost corner. Lengths of proportioned lines are comparable only when reduced to their cardinal equivalents. The method may be referred to as a four-way proportion.

The record measurements must be reduced to their cardinal equivalents. The cardinal equivalent for a north and south line is the record distance times the cosine of the bearing, and for an east and west line it is the record distance times the sine of the bearing.

7-9. Use of cardinal equivalent employs only the northerly components (latitudes) of the north and south controlling record lines to compute the latitudinal position, and only the easterly components (departures) of the east and west controlling lines to compute the longitudinal position. This is different from using distances of the controlling record lines in the computation of proportionate measurement.

Figure 7-1 from the Manual illustrates the plan of double proportionate measurement. Points A, B, C, and D represent four original corners that control the restoration of the lost corner X. On the large-scale diagram the point E represents the proportional measurement between A and B, and similarly, the point F represents the proportional measurement between C and D. The point X is at the latitude of E and at the longitude of F. The directions from E to X and from F to X are cardinal.
Figure 7-2 from the Manual shows the calculations for double proportion. A temporary point is established in the vicinity of the lost corner and measurements are made to the controlling corners. Then offsets from the temporary point to the true corner point are calculated. The method requires that the basis of bearings for measurements be true. This can be accomplished by making an astronomic observation for true meridian at the temporary point, or by rotating a state plane coordinate system survey to the true meridian. See Appendix A for an explanation of the calculations.

Figure 7-2. Double proportionate measurement and cardinal equivalents.
Double Proportion Example Using State Plane Coordinates

The example is in Section 16, T30N, R44E, W.M., Pend Oreille County, Washington. The basis of bearings and coordinates is the Washington Coordinate System, North Zone, NAD83 (1991). The survey computes a double proportion position for corner N9, the corner of sections 16, 17, 20 and 21.

Here the cardinal equivalents of the record measurements are little different from the total measured distances. They should always be calculated, nevertheless, so that they will not be ignored in those cases where they do make a difference.

Cardinal Equivalents of East-West Lines:
- N89°46’W, 80.34 chs: Lat. +0.3272, Dep. -80.3393
- N89°50’W, 40.23 chs: Lat. +0.1170, Dep. -40.2298
Total Departure for E-W proportioning: 120.5691 chs.

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10 Book 3 of Surveys, Pages 153 to 156, records of Pend Oreille County, Washington, recorded 11 October 1994.
Cardinal Equivalents of North-South Lines:
N00°04’W, 80.00 chs:  Lat. +79.9999,  Dep. -0.0931
N00°04’W, 40.00 chs:  Lat. +40.0000,  Dep. -0.0465
Total Latitude for N-S proportioning:                  119.9999 chs.

Coordinates of controlling corners:
L9:  420362.066 / 2512679.621
N7:  417568.444 / 2510152.087
N13: 417924.181 / 2518100.550
R9:  412431.941 / 2513026.260

Calculate the temporary positions along the lines connecting the controlling corners:
L9 to R9     = S 2°30'10"E 7937.697 feet measured, 120.000 chains GLO cardinal equivalent.
N7 to N13 = N87°26'15"E 7956.420 feet measured, 120.569 chains GLO cardinal equivalent.

North/South proportion = 7937.697 ÷ 120.000 = 66.1641 feet per chain.

East/West proportion = 7956.420 ÷ 120.569 = 65.9906 feet per chain.

Calculate temporary north and south point:
L9  420362.0660, 2512679.6210
    S 2°30'10"E 2645.899
E  417718.6912, 2512795.1673  <- temporary north south point
    S 2°30'10"E 5291.798
R9  412431.9410, 2513026.2600

Calculate temporary east and west point:
N7  417568.444, 2510152.087
    N87°26'15"E 2654.802
F  417687.142, 2512804.234 = temp east west point
    N87°26'15"E 5301.618
N13  417924.1810, 2518100.5500

Calculate convergence at the temporary points:
E     417718.69120 N
       2512795.16730 E
    Convergence 02 39 30.79525 Scale Factor 0.999942375
F     417687.142 N
       2512804.234 E
    Convergence 02 39 30.87829 Scale Factor 0.999942375
Calculate the grid direction of a cardinal offset at the temporary points:
The grid Azimuth = true azimuth minus convergence.
True north = grid 357°20’29’’ = N 2°39’31”W
In much of eastern Washington convergence is positive.
In all of western Washington convergence is negative.

Intersection of cardinal lines from temporary points to proportioned corner X:

<table>
<thead>
<tr>
<th>Pt</th>
<th>Bearing</th>
<th>Distance</th>
<th>Pt</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>N 2°39’31”W</td>
<td>31.936</td>
<td>Z</td>
</tr>
<tr>
<td>E</td>
<td>N87°20’29”E</td>
<td>7.594</td>
<td>Z</td>
</tr>
</tbody>
</table>

Double Proportion Solution:
Z = 417719.043 2512802.753
L9  to Z = S 2°40’02”E 2645.889
N7  to Z = N86°44’54”E 2654.941
N13 to Z = S87°46’57”W 5301.767
R9  to Z = N 2°25’14”W 5291.825
Three-Point Control

General Description:

7-13. Where the line has not been established in one direction from the lost township or section corner, three-point control should be used to reestablish the position of the corner. The record distances (reduced to its cardinal equivalent) to the nearest identified corner in the direction opposite from the missing line will be used, along with proportionate measurement between the other two corners.

The diagram\textsuperscript{11} above illustrates that three-point control is computed with cardinal offsets, just as in double proportion. One of the temporary points is at a record latitude from a single controlling corner.

\textsuperscript{11} Diagram taken from a BLM Advanced Cadastral Survey presentation by Mark Dixon, 1997, Renton, Washington
Three Point Control Example

Below is the GLO plat for a portion of T29N, R2W, W.M. The line between sections 27 and 34 was measured only in part. There is no record total distance for the line between sections 27 and 34. The section line monument 35.00 chains west of the corner of sections 26, 27, 34, and 35 is lost. The restoration of the section corner of sections 26, 27, 34 and 35 must be by three-point control using the nearest found corners north, east and south. See Appendix B for the calculations.
Two Point Control

General Description:

7-14. Where the intersecting lines have been established in only two of the directions, two-point control should be used to reestablish the position of the corner. The record distances, reduced to their cardinal equivalents, to the nearest identified corners on the intersecting lines will control the position of the corner. ... What is intended by record distance is the measure established by the original survey.

The figure\textsuperscript{12} above illustrates that two-point control is computed with cardinal offsets just as in double proportion. Both temporary points are at a record latitude or departure from the two controlling corners.

Index Correction:

7-15. An index correction for systematic error in measurement should be made in applying the record measurements for two or three-point control (section 7-57) if it is obvious that a more harmonious relation to the representations of the approved plat or plats would be thus accomplished. ... It is only a demonstrable and consistent excess or deficiency of the original work, determined within practical limits, that can justify the application of an index correction. If such consistency is not established the only rule that can be applied is that a record of 80.00 chains in distance means just that by exact standards, true horizontal measurement.

Apply an index correction only after measuring many miles of lines in the township and determining that there is a consistent scale and/or rotation factor between the GLO survey and modern, more accurate measurements.

Single Proportion

General Description:

7-16. The term single proportionate measurement is applied to a new measurement made on a line to determine one or more positions on that line. By single proportionate measurement the position of two identified corners controls the direction of that line. The method is sometimes referred to as a two-way proportion. Examples are a quarter-section corner on the line between two section corners, all corners on standard parallels, and all corners occupying intermediate positions on a township boundary line.

7-17. In order to restore a lost corner on a line by single proportionate measurement, a retracement is made connecting the nearest identified corners on the line. These corners control the position of the lost corner. The lost corner is then reestablished at proportionate distance on the line connecting the recovered corners. Proper adjustment is made on an east and west line to secure the latitudinal curve. Any number of intermediate lost corners may be located on the same plan.

Single Proportion is used in more situations than other proportioning methods. It is used for standard parallels, base lines, township boundaries, junior and senior corners of a line, closing corners, quarter-section corners, and some meander corners. See Appendix C for more quotations from the Manual regarding single proportion.

BLM surveys adjust for the latitudinal curve on east and west lines. For example, the center of section, if set on a BLM survey, will not be on a line of sight between the east and west quarter-section corners. It will be on a straight line that follows the latitudinal curve. Non-federal surveys usually represent section subdivision boundaries as lines of sight instead of straight lines and accommodate the latitudinal curve only along east and west township boundaries, base lines, and sectional correction lines in those instances where long distances between controlling corners require offsets to the latitudinal curve in order to avoid large errors.
Single Proportion Example

Sections 3 and 4, T34N, R38E, W.M.
In this survey there was a lost closing corner between sections 3 and 4. This is not a standard parallel. The closing corners were needed because of irregularities in the previous GLO survey.

![Survey Diagram]

Closing Corners

The survey found the section corners for section 34, which is north of section 3. The quarter-section corner on the south line of section 34 was lost. The total GLO distance for the south boundary of Section 34 is 80 chains. The survey measurement is 5298.87 feet. The proportion of feet to chains is $5298.87 / 80 = 66.2358750$ feet per chain. The proportion converts the record measurements to measurements of the new restoration survey.

The GLO set the closing corner 9.10 chains east of the southwest corner of section 34. Therefore the single proportioned position is $(9.10 ÷ 80) * 5298.87 = 602.75$.

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The same survey illustrates a peculiarity of closing corners as described in the 2009 Manual. Closing corners are the only original federal corners subject to amendment.

7-45. When an original closing corner is recovered off the line closed upon and the new monument is established at the true point of intersection, the original position will control in the proportionate restoration of lost corners dependent upon the closing corner. In a like manner the positioning of sixteenth-section corner(s) or lot corner(s) on the closing line, between the quarter-section corner and the closing corner, will be based on the measurement to the original position of the closing corner.

The closing corner of sections 4 and 5 was found to be 5.70 chains feet south of the township line. The true closing corner was established by extending the section line north 5.70 feet to meet the township line.

The north sixteenth-section corner of sections 4 and 5 is proportioned using the original GLO closing corner as control, not the true closing corner.

The GLO distance from the quarter-section corner to the closing corner is 39.00 chains. The measured distance is 2572.73 feet.
The proportioned distance from the quarter-section corner to the north sixteenth-section corner is 
\[(20.00 \div 39.00) \times 2572.73 = 1319.35 \text{ feet.}\]

The proportioned distance from the north sixteenth-section corner to the original closing corner is 
\[(19.00 \div 39.00) \times 2572.73 = 1253.38 \text{ feet.}\]

The total distance from the north sixteenth-section corner to the true closing corner on the township line is 
\[1253.38 + 5.70 = 1259.08 \text{ feet.}\]

**Quarter-Section Corner Not Established**

The survey above also is an example of the fact that the closing quarter-section corners of sections with closing corners was not usually established by the GLO survey.

7-46. *The quarter-section corners for sections on the side to which the closing corners refer were often not established in older surveys. The correct positions are protracted on the plat of these sections. When a new monument is to be established at the protracted quarter-section position, the original position of the section closing corners will control in the establishment. The proportionate measurement position between the original positions of the section closing corners will be moved in a cardinal direction ... to the true point on the line. This true point for the quarter-section corner will control the location of the legal subdivisions for the section on the side to which the closing corners refer.*

Since the original closing corner of sections 3 and 4 is lost, the north quarter-section corner of section 4 is calculated by establishing a midpoint between the original closing corner of sections 4 and 5 and the proportioned closing corner of sections 3 and 4. The midpoint position is moved in a cardinal direction to the township line to establish the true point for the quarter-section corner. Before the 2009 Manual there was not a prescribed procedure for establishing such a quarter-section corner and other methods were sometimes employed.\(^{14}\) In this case, in 1994, the

\(^{14}\) The two most common methods formerly employed that differ from the 2009 Manual method are (1) midpoint between the true closing corners and offset to the township line in a cardinal direction, (2) midpoint between the original closing corners and establish the quarter-section corner where the line connecting to south quarter corner intersects with the township line.
offset to the township line was not cardinal. It was along the line connecting the south quarter-section corner with the midpoint between closing corners. Below are the calculations using a cardinal offset. The coordinate system is Washington State Plane, North Zone, NAD 83 (1991), US Survey Feet.

Calculate temporary midpoint between closing corners:
324 552644.313 2317278.798 Original Closing Corner Secs 4 & 5
    N88°01'48"E 2651.795
1001 552735.468 2319929.025 Temp mid-point between closing corners
    N88°01'48"E 2651.795
801 552826.624 2322579.253 Proportioned Closing Corner Secs 3 & 4 (original is lost)

Calculate the grid bearing for cardinal offset to township line:
Convergence: +2°05'10.91"
True North:    N2°05'11"W

Offset to township line:
1001 552735.468 2319929.025 Temp mid-point between closing corners
    N2°05'11"W 4.195
1002 552739.661 2319928.873 True North Quarter-Section Corner Sec 4
    S88°13'02"W 603.259
334 552720.892 2319325.906 South Quarter-Section Corner Sec 33

The distance from the calculated north quarter-section corner of section 4 to the south quarter-section corner of section 33 is 0.03 feet shorter than the distance shown on the recorded survey map. Using the bearing of the line to the south quarter section corner instead of a cardinal bearing for the offset to the township line caused an error of three hundredths of a foot.

This same section 4, T34N, R38E, is discussed further in the chapter on section subdivision.
Single Proportion Along a Township Line

The Manual requires adjustment on an east and west line to secure the latitudinal curve. Such a rhumb line is called a *straight line* by the BLM. A way to understand the “straightness” of such a line is to imagine following a well adjusted compass in an east and west direction. The compass would direct you to follow just such a “curved” line but you would have the perception of following a “straight” line.

An example is a survey\(^{15}\) of the Offset Second Parallel North along the south boundary of T10N, R39E. W.M. Due to the length of the township line between controlling corners, it is shown below in two figures.

There is extensive loss of corners along the standard parallel. The survey goes through rolling wheat fields. The west sixteenth-section corner between sections 34 and 3 is a preserved position of an ancient fence corner that had already been held to be a valid monument of the standard parallel and had been used to proportion the section corner to the east. The sixteenth-section corner was held as the westerly control for proportioning and the existent quarter-section corner of sections 1 and 36 was held for the easterly control.

The proportioned quarter-section and section corners are adjusted for the latitudinal curve. They are on a *straight line*. On the BLM survey the bearings between the several corners of the line would be the same bearing. However, since this survey employs the Washington Coordinate System, South Zone, NAD 83 (1991), each bearing is the grid bearing of the *line of sight* between the corners, which is a different bearing in each case due to the changing convergence angle as the survey moves east and west. Appendix D shows three different ways to calculate the single proportions: (1) using the BLM program WinCMM, (2) using a formula for the offset to the latitudinal curve developed by Elgin, Knowles & Senne and publish many years ago in the trade magazine, *Point of Beginning*, and (3) converting the state plane coordinates to geographic coordinates, proportioning the latitudes and longitudes for the intervening corners, and then converting back to state plane coordinates.

\(^{15}\) See Book 8 of Surveys, Pages 71 to 73, records of Columbia County, Washington, 23 June 2008.
Irregular Boundary Adjustment

General Description:

7-51 & 7-52. Some township boundaries are not established as straight lines and are termed “irregular” exteriors. … A modified form of single proportionate measurement is used in restoring lost corners on such boundaries. This is also applicable to a section line or a township line that has been shown to be irregular by a previous retracement. … The adjustment to be applied along the line is single proportion, and the adjustment to be applied perpendicular to the direction of the line is compass rule. On a meridional line the latitude of the closing distance, presuming retracement is made on record courses and distances, is distributed among the courses in proportion to the latitude of each course. The departure of the closing distance is distributed among the courses in proportion to the length of each course. … On a latitudinal line …

Irregular Boundary Adjustments restore lost corners along lines that would normally be straight lines but are shown on federal survey plats to have angle-points. The adjustment is a mixture of single proportion and compass rule adjustment of the record courses to fit found controlling corners of the line. As noted in the Manual, the adjustment method can only be applied when a “retracement is made on record courses and distances.” If the adjustment is made on an east and west line adjustments to the latitudinal curve must be applied. There is an example of such an adjustment to an east and west line in Appendix E.

The adjustment of an irregular north and south line is simpler because adjustments to the latitudinal curve can be ignored. However, the requirement to adjust a retracement “made of record courses and distance” must be adhered to in order to properly differentiate between the different adjustment methods for true latitudes and for true departures.
Irregular Boundary Adjustment Example

Below is a GLO survey plat showing the irregular boundary retracement of the range line between T33N, R9E, and T33N, R10E. The survey accepted a found monument at the west quarter-section corner of section 31, an angle-point of the range line. A proportioned position was calculated for the corner in order to test the position of the found monument.

The southeast corner of section 36 (Z25) and the standard section corner of sections 30 and 31 (V1) were found. The coordinate system of the survey is the Washington Coordinate System, North Zone, NAD 83 (1991). The combined scale factor is 0.99994797.

There are two calculation methods shown here. These two methods should not be used on an east and west irregular boundary because they do not adjust for the latitudinal curve. See Appendix E for an irregular east and west boundary that is calculated using the BLM program WinCMM.

First calculate the convergence angle at the southeast corner of section 36.

Z25  474704.2937 N  1459287.4226 E

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16 See Book 41 of Surveys, Pages 161 to 170, records of Snohomish County, Washington, 12 November 1993, and Book 20 of Surveys, Pages 19 to 21, records of Skagit County, Washington 15 December 1997.
Convergence: -00 33 14.56976 Scale Factor: 0.999947237

Inverse between the found corners.
Z25  474704.2934 1459287.4226
     N 2°42'47"E 4679.271 <- grid inverse
V1  479378.3195 1459508.9215

Rotate -0°33'15", holding the coordinate at Z25, to make the basis bearings true at corner Z25.
Z25  474704.2934 1459287.4226
     N 2°09'32"E 4679.271
V1:1 479380.2431 1459463.7045

Run the GLO record from Z25 to V1, using the combined scale factor of 0.99992952.
Here the record distances are adjusted to grid instead of adjusting the measured distances to ground distances and then adjusting back to grid distances at the end. The final answer is the same either way.
Z25  474704.2934 1459287.4226      N 1°53'00"E 2048.500 (31.04 chains)
a     476751.6868 1459354.7455          N 2°33'00"E 2644.430 (40.07 chains)
b     479393.4982 1459472.3993          S 0°00'00"E 13.255 (northing misclosure)
c     479380.2431 1459472.3993          S90°00'00"W  8.695 (easting misclosure)
V1:1 479380.2431 1459463.7045

Distribute the latitudinal (northing) error based on the latitude (northing) of each course.
Total latitude: northing of “b” minus northing of Z25 = 4689.2048
1st course: northing of “a” minus northing of Z25       = 2047.3934
2nd course: northing of “b” minus northing of “a”       = 2641.8114
Adjustment to northing of “a” : -(2047.3934 ÷ 4689.2048 * 13.255) = -5.787
Adjustment to northing of “b” : -13.255

Distribute the departure (easting) error based on the length of each course.
Total length: 2048.500 + 2644.430 = 4692.930
Adjustment to the easting of “a” : -(2048.500 ÷ 4692.930 * 8.695) = -3.795
Adjustment to the easting of “b” : -8.695

a:  476751.687 – 5.787 =  476745.900
     1459354.745 – 3.795 = 1459350.950
b: = V1:1

Adjusted GLO record
Z25:1 474704.2934 1459287.4226      N 1°46'56"E 2042.595
a     476745.9000 1459350.9500          N 2°27'03"E 2636.755
V1:1  479380.2431 1459463.7045

Rotate 0°33’15” at Z25 to make bearings grid.
Z25:1 474704.2934 1459287.4226      N 2°20'11"E 2042.595
a     476745.1901 1459370.6932          N 3°00'18"E 2636.755
V1:1  479378.3195 1459508.9215 = V1 the measured position of the corner

Alternatively, one can rotate the GLO record to grid and employ a compass adjust. This is not the correct method but, when the line does not have very large angle points, the answer will be close to the answer produced by the correct calculation.

Rotate the GLO record +0°33’15” at Z25 to make bearings grid.

N 1°53'00"E 2048.500 (31.04 chains)  rotated is N 2°26'15"E
N 2°33'00"E 2644.430 (40.07 chains)  rotated is N 3°06'15"E

Raw Traverse:
Z25  474704.2934 1459287.4226  N 2°26'15"E 2048.500
x  476750.9399 1459374.5445  N 3°06'15"E 2644.430
y  479391.4898 1459517.7441  S33°49'03"W  15.852 <- misclosure
V1  479378.3195 1459508.9215

Balanced by the compass rule:
Z25  474704.2934 1459287.4226  N 2°20'11"E 2042.596
x  476745.1909 1459370.6933  N 3°00'18"E 2636.754
V1  479378.3195 1459508.9215

The irregular boundary adjustment procedure and the compass adjust produce the same results.
Irregular boundary adjustment:  a:  476745.1901 1459370.6932
Compass rule adjustment:  x:  476745.1909 1459370.6933
Angle Points of Meander Lines

General Description:

7.53. In some cases it is necessary to restore (or possibly to locate for the first time) the angle points, within a section, of the record meander courses for a stream, lake, or tidewater, required under the special rules applicable to meander lines. This is commonly called the compass rule adjustment. … The general rule is that the adjustment to be applied to the latitude or departure of any course is to the resolved latitude or departure of the closing error as the length of the course is to the total length of all the courses.

The compass rule yields the same results no matter what the meridian or distance scale factor. It is essential to ensure that the record measurements and the retraced measurements have the same basis. This is most easily accomplished by rotating the GLO record bearings, at the point of beginning of the meander line, and scaling the GLO record distances to the basis of the current survey. This method is technically incorrect because it does not account for the latitudinal curve and for converging meridians. The error is greater when there is a significantly different longitude between beginning and end points of the meander line. In such a case a more valid adjustment can be made by adjusting the rotation of the record GLO bearing to grid individually for each course based on the convergence angle in the middle of each course. This will result in the rotated GLO record having different angles between the line of sight bearings at the rotated angle points of the meander line than the angles between the straight line bearings at the angle points of the GLO record.
Meander Line Adjustment Example

The GLO survey plat of Section 16, T16N, R11W, W.M.

The GLO record measurements along the west boundary of section 16
The survey\textsuperscript{17} was done in the Washington Coordinate System, South Zone, NAD 83 (1991) with average combined scale factor of 0.9999302.

First enter the record courses using GLO bearings and distances, starting at the meander corner of sections 16 and 21, scaled by the combined scale factor of 0.9999302.

<table>
<thead>
<tr>
<th>No.</th>
<th>X</th>
<th>Y</th>
<th>Bearing</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>579067.8960</td>
<td>750488.6570</td>
<td>N33°00'00&quot;E</td>
<td>1451.900</td>
</tr>
<tr>
<td>24</td>
<td>580285.5618</td>
<td>751279.4184</td>
<td>N20°00'00&quot;E</td>
<td>857.940</td>
</tr>
<tr>
<td>25</td>
<td>581091.7617</td>
<td>751572.8512</td>
<td>N10°00'00&quot;E</td>
<td>1715.880</td>
</tr>
<tr>
<td>26</td>
<td>582781.5736</td>
<td>751870.8106</td>
<td>N20°00'00&quot;E</td>
<td>1451.900</td>
</tr>
<tr>
<td>27</td>
<td>584145.9133</td>
<td>752367.3897</td>
<td>N 5°00'00&quot;E</td>
<td>224.380</td>
</tr>
<tr>
<td>28</td>
<td>584369.4395</td>
<td>752386.9457</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Next rotate the traverse by the convergence angle at the meander corner between sections 16 and 21.

<table>
<thead>
<tr>
<th>MC 16/21</th>
<th>X</th>
<th>Y</th>
<th>Bearing</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>579067.8960</td>
<td>750488.6570</td>
<td>N35°35'07&quot;E</td>
<td>1451.900</td>
</tr>
<tr>
<td>24</td>
<td>580271.7239</td>
<td>751311.0690</td>
<td>N21°25'38&quot;E</td>
<td>865.604</td>
</tr>
<tr>
<td>25</td>
<td>581077.4997</td>
<td>751627.2907</td>
<td>N11°32'04&quot;E</td>
<td>1736.995</td>
</tr>
<tr>
<td>26</td>
<td>582779.4162</td>
<td>751974.6139</td>
<td>N21°25'38&quot;E</td>
<td>1464.870</td>
</tr>
<tr>
<td>27</td>
<td>584143.0382</td>
<td>752509.7589</td>
<td>N 6°35'59&quot;E</td>
<td>227.489</td>
</tr>
<tr>
<td>15</td>
<td>584369.0200</td>
<td>752535.9050</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Next, adjust using the compass rule.

<table>
<thead>
<tr>
<th>No.</th>
<th>X</th>
<th>Y</th>
<th>Bearing</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>579067.8960</td>
<td>750488.6570</td>
<td>N34°20'22&quot;E</td>
<td>1457.931</td>
</tr>
<tr>
<td>24</td>
<td>580271.7239</td>
<td>751311.0690</td>
<td>N21°25'38&quot;E</td>
<td>865.604</td>
</tr>
<tr>
<td>25</td>
<td>581077.4997</td>
<td>751627.2907</td>
<td>N11°32'04&quot;E</td>
<td>1736.995</td>
</tr>
<tr>
<td>26</td>
<td>582779.4162</td>
<td>751974.6139</td>
<td>N21°25'38&quot;E</td>
<td>1464.870</td>
</tr>
<tr>
<td>27</td>
<td>584143.0382</td>
<td>752509.7589</td>
<td>N 6°35'59&quot;E</td>
<td>227.489</td>
</tr>
<tr>
<td>15</td>
<td>584369.0200</td>
<td>752535.9050</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{17} See Book 18 of Surveys, Pages 13 and 14, records of Grays Harbor County, Washington, 20 May 1997.
Grant Boundary Method

General Description:

7-54. In many of the States there are irregular grant and reservation boundaries that were established prior to the public land rectangular surveys. … This is essentially a rotate and scale procedure.

No example calculations of the Grant Boundary Method are given here.

In addition to reservation boundaries, the method might be applicable in some adjustments of mineral surveys and Donation Land Claim lines.
**No Hard and Fast Rule**

For reestablishing lost corners in the following types of federal surveys there is no hard and fast rule:

- Donation land claims
- Homestead entry surveys
- Mineral surveys
- Tract corners
- Townsites

The 2009 Manual instructions for such retracements:

7-55. *There is no hard and fast rule for reestablishing lost corners of tract surveys and special surveys (see section 10-213 for lode mining claim surveys). When the original surveys were made faithfully, the application of the principles of record distances, record angular relationships, and record relationships between improvements and adjoining surveyed parcels, in combination with the presumption that the original intent was to be conformable with the statutes governing orientation, dimensions, and area, will substantially meet the objects stated above.*
One-Point Control

General Description:
7-56. Where a line terminates with measurement in one direction only, a lost corner should be restored by record bearing and distance, counting from the nearest identified or restored regular corner. Examples will be found where lines have been discontinued at the intersection with large meanderable bodies of water or at the border of what was classed as impassable ground. ... An index correction should be applied to the record bearing and/or distance when applicable.

Index Correction:
7-57. If there is no conclusive evidence of applicability of an index correction, the record courses and distances should be allowed to prevail.

The figure\textsuperscript{18} below illustrates restoration of a meander corner by one-point control.

\begin{center}
\begin{tikzpicture}
\draw[->,thick] (0,0) -- (2,0) node[midway, above] {WEST 75.40};
\draw[thick] (0,0) -- (0,1) node[midway, left] {MC};
\draw[->,thick] (0,0) -- (1,1) node[midway, right] {35.40};
\draw[->,thick] (0,0) -- (1,-1) node[midway, right] {40.00};
\end{tikzpicture}
\end{center}

To restore the lost MC and 1/4 cor. use "1 Point Control" which is record bearing and distance.

\textbf{CAUTION:} If you are not on the same basis of bearing (true mean astronomic) as the record, you may end up in the wrong place.

When the line to the meander corner is at the terminus of a north and south line, a rotation and scale factor for the GLO record measurement can be calculated at the beginning of the line. When the meander corner is at the terminus of an east and west line, care must be taken to rotate the GLO record by a rotation angle calculated at the middle of the line to the meander corner. In that manner the plane coordinate system \textit{line of sight} bearing will be an accurate reproduction of the bearing of the GLO \textit{straight line} record bearing.

One-Point Control Example

The survey, in section 32, found both meander corners to be lost, on the east and west sides of the section. The coordinate system for the survey was the Washington Coordinate System, North Zone, NAD 83 (1986) with a combined scale factor of 0.9999361.

First find the convergence angle and scale factor at the two controlling section corners.

\[
\begin{align*}
Z5 & \quad 418066.52830 \ N \quad 1149715.80420 \ E \quad \text{SW section corner} \\
\text{Convergence} & \quad -01 \ 29 \ 46.4221 \\
\text{Scale Factor} & \quad 0.999942271 \\
Z9 & \quad 417920.73130 \ N \quad 1155013.89900 \ E \quad \text{SE section corner, calculated in lake} \\
\text{Convergence} & \quad -01 \ 28 \ 48.01230 \\
\text{Scale Factor} & \quad 0.999942271
\end{align*}
\]

The elevation scale factor is 0.9999938.
The combined grid scale factor is 0.9999361.

From Z5 the meander corner is N 1°29’46”E, 1029.53 feet (15.60 chains)
\[0.9999361 \times 15.60 \text{ chains} \times 66 \text{ ft/ch} = 1029.53.\]

From Z9 the 1/4 corner, X9, is N 1°28’48”E, 2639.83 feet (20.00 chains)
\[0.9999361 \times 20.00 \text{ chains} \times 66 \text{ ft/ch} = 2639.83.\]

From the 1/4 corner, X9, the meander corner is N 1°28’48”E 329.98 feet (5.00 chains)
\[0.9999361 \times 5 \text{ chains} \times 66 \text{ ft/ch} = 329.98.\]

Notice that the bearings to the meander corners converge towards each other.

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\[19 \text{ See Book 17 of Surveys, Pages 65 to 69, records of Jefferson County, Washington, 14 September 1995.} \]
Section Subdivision

From the Specimen Plat in the pocket of the 2009 Manual
Names of Subdivision Corners

Subdivision of sections will be discussed to the quarter-quarter-section, or sixteenth-section, level. Subdivision to lower levels follow the same principles. The names of the corners of quarter-quarter sections, or sixteenth sections, should be familiar.

The names written out are as follows:
- Center quarter corner of section 36
- West sixteenth corner of sections 25 and 36
- Northeast sixteenth corner of section 36
- Center-east sixteenth corner of section 36

These names are often abbreviated as follows:
- C 1/4 corner of section 36
- W 1/16 corner of sections 25 and 36
- NE 1/16 corner of section 36
- C-E 1/16 corner of section 36
Found Monuments of Local Origin

It is necessary, when subdividing sections, to know what to do with monuments established by previous surveys. Monuments of unknown origin, if accepted, are treated as if they were established by a previous survey.

That federally established original corner monuments have high authority is clear from the federal laws mentioned above. The authority of locally established original corners, however, and the authority of local retracements of previous surveys are not so well defined. There are four categories of found monuments of local origin set by authorized surveys.

Firstly, there are original corner monuments set by local (state authorized) surveyors in the course of creating new platted areas or lots under the authority of state, county, and city platting laws. These monuments have wide acceptance as not being subject to rejection or modification, much like federally established section and quarter-section corners.

Secondly, there are original corner monuments set by local surveyors while subdividing federally surveyed sections. There are federal rules and procedures for subdivision of sections. Those rules, as revealed in Federal laws and BLM manuals and circulars, have been adopted by law in Washington as legal section subdivision procedures. Federal law gives local surveyors the authority to set original section subdivision corners. Washington State law gives local registered surveyors the authority to establish original corners. The 2009 Manual, authorized for use by Washington law, sets up restrictions to the ability to reject such corner monuments. Sometimes those corners, established with the proper authority, have been rejected by registered surveyors. That rejection is usually for one of two primary reasons. The corner may be judged to have been set using improper methods or procedures. An example would be the rejection of a center quarter-section corner monument because it was established using an incorrect monument for one of the four controlling quarter section corners. The second source of corner monument rejection is the establishment of a monument at a location incorrect mathematically to such a degree that the corner is deemed unacceptable. For example, a center quarter-section corner may have been set using the wrong backsight with the result that it is tens of feet from where it was intended to be. These two criteria are not used to reject an original corner monument set by a federal surveyor, although the criteria have been applied to federal dependent resurvey proportionate restorations. Rejection based on the first criterion, improper methods or procedures, is valid; it has been employed by federal surveys in the rejection of local corners. Rejection based on the second criterion, inaccurate monument location, must be tempered by an evaluation, probably subjective, of the required positional accuracy.

Thirdly, there are corner monuments set to replace missing monuments set in the course of creating platted areas or lots under the authority of state, county, and city platting laws. Such corner monuments should be established in the location of the original monument and, if the original location is lost, the monument must be established using some sort of proportioning technique. Monuments having satisfied that requirement are then judged based on the accuracy of the retracement position.

---

20 See 2009 Manual section 3-137 quoted above in discussing the “local surveyor.”
Fourthly, there are corner monuments set to replace missing monuments set by local surveyors while subdividing federally surveyed sections. Retraces of previously surveyed section subdivision corners are sometimes treated differently from retraces of corners established under state and local platting authority. Previously established section subdivision corners, if lost, are sometimes replaced by corner monuments set at theoretical locations calculated from the controlling section and quarter-section corners. If the original location of the section subdivision corner can be determined from nearby references or other means, then the corner monument should be reestablished in the original location, not in a new location calculated from ties to section and quarter-section corners. Such respect for the original locations of corner monuments stabilizes land boundaries.

Subdivision by Protracation and Subdivision by Survey

The 2009 Manual section after the header “Subdivision of Sections by Protraction” specify that the procedures are for the federal draftsperson, not for the surveyor.

3-101. The following sections address the procedures to be followed by the draftsperson after receiving the field returns from the surveyor.

Some surveyors and some land surveying reference documents have advised following the methods of “subdivision by protraction” in a land survey. That is bad advice because the “subdivsion by protraction” procedures can violate federal law regarding section subdivision, especially in the case of fractional sections.

3-108. In the case of a section whose boundary lines are in part within the limits of a meandering body of water or within the boundaries of a private claim, the section lines are completed in theory and the protracted position of the subdivision-of-section lines is controlled by the theoretical points so determined.

Do not use the instructions in Manual section 3-108 for a field survey. Use instead the procedures under the header “Order of Procedure in Subdivision of Sections by Survey” beginning at Manual section 3-112.
Basic Procedure and Rules of Section Subdivision

The 2009 Manual summarizes the procedure of section subdivision.

3-113. The order of procedure is: First, identify or reestablish the marked corners on the section boundaries, including determination of the points for the necessary sixteenth-section corners. Next, fix the boundaries of the quarter sections and then form the quarter-quarter sections or lots by equitable and proportionate division.

The basic rules for subdividing a section into 1/16 sections:
1. The center of section, or center quarter-section corner, is at the intersection of straight lines connecting opposite quarter-section corners.
2. When no opposite quarter-section corner can be fixed, survey on a mean or parallel bearing.
3. A sixteenth-section corner is:
   • At the midpoint between controlling corners when there are no lots involved and the closest controlling corners are section or quarter-section corners.
   • At a proportioned distance between controlling corners when there are lots involved.
   • At a proportioned distance when a controlling corner is not a quarter-section or section corner, for example a meander corner or a line tree.
4. The center of a quarter section is at the intersection of straight lines connecting opposite sixteenth-section corners.
5. When no opposite sixteenth-section corner can be fixed, survey on a mean or parallel bearing.

An acceptable local corner will have precedence over a location calculated using these rules.
Types of Sections

There are three types of section subdivision:
- Regular Section Subdivision
- Fractional Section Subdivision
- Closing Section Subdivision (a variation on regular subdivision)

The 2009 Manual makes some additional distinctions between types of sections.

Regular Sections

General description:
3-32. The regular township includes 36 sections in all, 25 of which are regular sections returned as containing 640 acres each, subdivided into regular aliquot parts, based on midpoint protraction and intersections. Irregular sections against the north and west boundaries ... contain regular aliquot parts ... with ... additional regular lots, each containing 40 acres plus or minus the excess or deficiency in measurement. ... The aforementioned section returned as containing 640 acres is termed “regular” with aliquot part legal subdivisions, such as a half-section, a quarter-section, a half-quarter section, or a quarter-quarter section.

All other sections are irregular sections.

This township contains 25 Regular Sections
Invaded Sections - Irregular

Invaded Sections are subdivided into sixteenth-sections using either the methods of regular section subdivision or using the methods of fractional section subdivision, depending on how the section was created by federal survey.

Definition:

3-106. Sections that are invaded by meanderable bodies of water or by approved claims at variance with the regular legal subdivisions ...

An Invaded Section from the Manual Specimen Plat

Fractional Sections - Irregular

A fractional section is one of two types:

3-118. By law a fractional section is (1) a section containing outlying areas protracted as surveyed, or (2) an invaded section in which at least one quarter-quarter section corner has not been or cannot be fixed.

Definition of a corner that is not fixed:

3-119. The law presumes that a corner has not been fixed when: (1) the section line on each side of the corner position has not been actually run, or (2) the section line has been actually run but at least one corner on either side, on the section line at issue, has not been monumented. The rule presumes that a section line has been actually run when a bearing and distance of the line is returned in the official survey record.

If a corner does not meet these criteria for not being fixed then it is a “fixed” corner whether it has been monumented or not. In section 12 from the Specimen Plat, shown above, the quarter-
section corner that was not monumented in the rancho was fixed by the measurement across the rancho.

Example of an Outlying Areas Fractional Section where there are protracted and platted areas lying outside of the surveyed section lines

Example of an Invaded Fractional Section where the section line on each side of the theoretical north quarter-section corner has not been actually run

Figure 3-44. Fractional section. No bearing and distance returned between the meander corners.
Example of an Invaded Fractional Section where the section line through the theoretical east quarter-section corner has been run but at least one corner, the northeast section corner, on either side of the theoretical east quarter-section corner, has not been monumented.

Below is an Invaded Section from the Specimen Plat that does not meet the definition of a Fractional Section because the west quarter-section corner is fixed by measurement across the river. Some surveyors in the past have treated this type of section as fractional for subdivision purposes. Older surveys that have subdivided such sections as fractional should be given some recognition.
Example of an Invaded Fractional Section caused by an Indian Reservation Boundary
Sec 33, T21N, R11W, W.M.
Closing Sections - Irregular

The 2009 Manual does not define closing sections. They normally occur along the north and west boundaries of a township and contain lots along those boundaries where the excess or deficiency in distance and acreage in the township is accumulated. In the case of townships much larger than normal, more than one series of lots can occur against the township boundary. Such sections are called elongated sections, and are subdivided in the same manner as other closing sections.

A Closing Section on the Manual Specimen Plat

Closing sections can occur along other township boundaries that are found to be defective. The figure below from the 2009 Manual shows a township with closing sections along all four exteriors.

Figure 3-30. Projection of both the sectional guide meridian and sectional correction line where combination of defective conditions exists.
Regular Section Subdivision

A Regular Section from the 2009 Manual Specimen Plat

Center of the section in a regular section:

3-114. To subdivide a regular section into quarter-sections, run straight lines from the fixed quarter-section corners to the opposite corresponding quarter-section corners. The point of intersection of the lines thus run and fixed will be the corner common to the several quarter-sections, or in other words, the legal center of the section.

The instructions are to create quarter sections using straight lines, i.e. using lines of constant bearing. In practice local surveys rarely use straight lines but employ lines of sight, which in the case of a center of section at a latitude of 47 degrees introduces a positioning error of about 0.18 feet north of where the corner should have been monumented.

Subdivision of Quarter-Sections by Survey

3-117. Preliminary to the subdivision of quarter-sections, the quarter-quarter or sixteenth-section corners shall be fixed as nearly as possible equidistant or proportionate measurement from two corners which stand on the same line, and between the quarter-section corners and the center of the section. On the last half mile of the lines terminating on township boundaries, they should be placed at 20 chains, proportionate measurement, counting from the regular quarter section corner. Subsequent to the establishment of quarter-quarter or sixteenth-section corners, the center lines of the quarter-section shall be run as straight lines between opposite corresponding quarter-quarter or sixteenth-section corners on the quarter-section boundaries. The point of intersection of the lines thus run and fixed will be the legal center of a quarter-section.

Notice that the instructions apply to closing sections as well as regular sections.
Example of a subdivision of a regular quarter section into sixteenth-sections\textsuperscript{21}

\textsuperscript{21} See Book 67 of Surveys, Page 81, records of Clallam County, Washington, 6 January 2009.
Invaded Sections Subdivided as Regular Sections

The situations where invaded sections are subdivided using the methods of regular section subdivision are made clear by the definition of a corner that is not fixed, cited above in the definitions of types of sections. If the controlling corner for subdivision can be fixed, then use the regular section subdivision procedures.

The second specimen plat in 2009 Manual is a dependent resurvey and section subdivision. It shows a calculated quarter-section corner in a lake that is used to subdivide the section. The quarter-section corner can be calculated because the section line across the lake was measured.

2009 Manual Specimen Plat — Appendix II
The GLO plat of Section 22, T20N, R3W, W.M., shows an Invaded Section with Hamersley Inlet going through the middle creating lots on each side of the inlet. The east quarter-section corner was not monumented because the position fell inside the inlet.
The survey of section 22 proportioned the east quarter-section corner in the water and employed that position to calculate a center of section in the water. Then that center of section was used to subdivide the northeast quarter-section into sixteenth-sections.22

See Book 20 of Surveys, Pages 149 to 153, records of Mason County, 30 October, 1995. Note that the proportioning of the east quarter-section corner was not calculated between the meander corners, as is the current procedure, in order to respect the previous state authorized surveys that proportioned the quarter-section corner between section corners, a method that has at times in the past been recommended by the BLM.
Sections invaded by Donation Land Claims normally have had section and quarter-section corners established within the claims and can be subdivided using normal procedures. T14N, R2W, W.M.
Fractional Section Subdivision

Subdivide a fractional section by running mean or parallel courses:

3-120. The law provides that where no opposite corresponding quarter-section corners have been or can be fixed, the subdivision-of-section lines shall be ascertained, by running a line from the monumented corners due north and south, or east and west, as the case may be, to the water-course, reservation line, or other external boundary of such fractional section, as represented upon the official plat. Under this subdivision-of-section method, the law presumes the section lines actually run and marked in the survey are due north and south, or due east and west lines, but usually this is not the case. Hence, in order to carry out the spirit of the law, it will be necessary in running the center lines through fractional sections to adopt mean courses, as ascertained from opposite corresponding section lines. Where an opposite corresponding section line does not exist, or the center line is platted parallel to one section boundary, run the center line parallel to the corresponding east, south, west, or north boundary of the section, as conditions may require.

Weighted mean mandated and defined:

3-121. The mean and parallel courses are based upon the weighted mean bearing of the controlling section line(s), equal to the bearing of the accumulated latitudes and departures of the controlling line(s).

Weighted Mean Illustrated, Figure 3-46 2009 Manual
The north and south centerline is the weighted mean of the east and west section lines.
The east and west centerline is a weighted mean of the south section line.

Problem: Compute the weighted mean bearing of the N-S center line of fractional section 20.
The data shown in diagram 1 is measured data.
Outlying Areas Fractional Sections

Above is part of an 1891 GLO survey in T30N, R5E, W.M. Sections 20, 21, and 28 have protracted platted outlying areas with some of the section lines and corners that should control the survey of the outlying aliquot parts left unsurveyed. The sections were later completed by GLO survey, but the areas platted by the 1891 GLO survey are established using only the lines and corners of the 1891 survey. On the line between sections 21 and 28, the east sixteenth-section corner was established by the 1891 GLO survey. Sections 20 and 21 were subdivided in 2009 and the boundaries were established as follows.23

- The west line of the northeast quarter of section 21 was surveyed on a bearing parallel to the north half of the east section line.
- The south line of the northeast quarter and the north line of the east half of the southeast quarter of section 21 was surveyed on a weighted mean bearing of the east half of the north section line and the east 20 chains of the south section line.
- Similar parallelism and weighted mean bearings were used in surveying the east half of the northeast quarter and the southeast quarter of section 28.

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23 See Book 1 of Surveys, Pages 61 and 62, AFN 2090201299, records of Whatcom County, Washington, 10 February 2009.
Invaded Fractional Sections

Section 16, T24N, R27E

The south line of section 16 above was surveyed only along the west 10 chains. The south quarter-section corner is not fixed by the GLO survey. The subdivision of section 16 established the north and south section centerline using a weighted mean bearing of the east and west section lines. 24 Similarly the north and south quarter-section centerlines in the south half of the section should be established using a weighted mean bearing of the east and west lines of the respective quarter-section.

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24 See Book 39 of Surveys, Pages 22 to 24, records of Grant County, Washington, 8 May 2000.
Fractional Quarter-Sections

Definition of a Fractional Quarter-Section:
3-123. By law, for subdivision purposes, a fractional quarter-section is within (1) a section containing outlying areas protracted as surveyed, or (2) an invaded section in which at least one quarter-quarter-section corner of the quarter-section has not been or cannot be fixed.

Subdivision of a Fractional Quarter-Section:
3-124. The subdivision-of-section lines of fractional quarter sections shall be ascertained by running from the properly established quarter-quarter or sixteenth-section corners with courses governed by the conditions represented upon the official plat. This can generally be accomplished by running due north and south, or east and west lines, as the case may be, to the watercourse, reservation line, or other external boundary of such fractional quarter-section.

In running the center lines through fractional quarter-sections it is necessary to adopt mean courses, as ascertained from opposite corresponding section and subdivision-of-section lines, or run parallel to a boundary of the section or quarter-section, as conditions require.

An invaded section that is not fractional clearly can have a fractional quarter-section. Section 22, T 20 N, R 1 W, W.M., is not fractional because all four quarter-section corners have been fixed. The southwest quarter-section of section 22 is fractional. The east west centerline of the southwest quarter-section must be run as a weighted mean bearing of the north and south boundaries of the quarter-section.

Section 22, T 20 N, R 1 W, W.M.
It is not clear whether certain quarter-sections are fractional or not. Section 17, T 19 N, R 3 W, W.M., below, provides an example.

Section 17 is not fractional because all four section corners exist and all four quarter-section corners can be fixed. But are any of the quarter-sections fractional, the northeast quarter-section, for example? The northeast section corner and the north quarter-section corner exist. The east quarter-section corner can be fixed by the section line having been measured. The center of section can be fixed from the four fixed quarter-section corners. The center-north sixteenth-section corner is fixed at midpoint between the quarter-section corner and the center of section. The west sixteenth-section corner on the north section line is fixed between the section corner and the quarter-section corner. But can the other two sixteenth-section corners around the northeast quarter-section be fixed? For the center-east sixteenth-section corner, the east quarter-section corner has been fixed but is not monumented in the water. For the north sixteenth section corner on the section line, the east quarter-section corner is fixed but not monumented in the water. The meander corner to the south of the north sixteenth is lost but can be monumented by calculation between the unmonumented east quarter corner and the section corner. The rule (Manual 3.119 above) states that a quarter-section corner that is not monumented is not fixed if one of the controlling corners on either side of the quarter-section corner is not monumented. Does the same rule apply in a scaled down fashion to a quarter-section? If one of the controlling corners for a sixteenth-section corner is fixed but not monumented, does that mean that the sixteenth-section corner cannot be fixed?
That question is, for now, left unanswered. Many past surveys have answered the question with a "No" and maintained that such a quarter-section is not fractional. Below is a non-fractional subdivision of the northeast quarter-section of Section 17 from a survey recorded in Mason County, Washington.
Closing Section Subdivision

The subdivision of closing sections presents some unique issues.

• Sixteenth-section corners along the closing line are not at midpoint between the quarter-section corner and the closing corner.
• The record distance from the sixteenth-section corner along the section centerline to the quarter-section corner on the township or range line is not shown on the GLO plat.
• Closing corners were not set along a retracement of the line closed upon and generally must be amended to the true point on the line closed upon.
• If there are closing corners, the quarter-section corner for the closing section along the township or range line that is closed upon often was not monumented by the GLO survey.

Closing Sections without Closing Corners

An example is section 6 from the 2009 Manual Specimen Plat.

The lengths of the east line of lot 1, of the north and west lines of lot 4, and of the south line of lot 7 are indicated on the plat. The distances are called parenthetical distances because they are shown in parentheses on the plat, indicating that the distance is not to a monument established on the ground. Older plats do not show the parenthetical distances, but they can be calculated from the total distance to the township line. The parenthetical distances are used to proportion the sixteenth-section corners along the closing lines. Two parenthetical distances, which are needed to subdivide section 6, are omitted from the plat: the distance between lots 2 and 3, and the distance between lots 5 and 6. The acreages shown on the plat are the key to determining those parentheticals.
Here are some GLO facts and calculation methods that can help in determining parenthetical distances.

One Acre = 10 Square Chains.

How to relate the side distances of a government lot to the acreage:

To compute the area of a lot that is 20 chains wide or high, add the two side distances, in chains, together. The reason for this is that every chain in height or width of the lot, along the side that closes into the township line, adds 2 acres to the size of the lot; 1 chain x 20 chains = 20 square chains = 2 acres. Twice the average height of the lot in chains is the acreage of the lot. The sum of the two side distances is twice the average height of the lot. Both lots in the figure below illustrate this.

How to relate the distance between two government lots to the acreages of the lots:

The distance of the line separating two lots, each 20 chains wide or high, is the sum of the two lot areas divided by four. This is because the area of a figure 40 chains wide is four times the average height of the area. The length of line dividing the area into two 20 chain wide lots is the average height of the two lots. See the figure below.

These calculations assume that the north boundaries of the two lots form a straight line.

Two rules for deriving parenthetical distances result from these area calculations.

- If the lot area and one side distance is known, subtract the known side distance from the area to get the other side distance.
- If the lot areas on both sides of a lot side line are known, divide the sum of the areas by four to get the distance of the line dividing the two lots.
In most closing sections with only four lots (those other than section 6) the distance between lots 2 and 3 is the mean of the lot side distances along the two section lines, such as in section 2 from the Specimen Plat below. The rules about the relationship of acreage to lot side distances will also give the correct answer for section 2.

Mean of lot distances along section lines: \( (20.00 + 20.01) \div 2 = 20.005 \)
Adding adjoining acreages and dividing by 4: \( (40.01 + 40.01) \div 4 = 20.005 \)

Using these rules one can calculate the distances between lots 2 and 3 and between lots 5 and 6 in section 6 below. The distance between lots 2 and 3 were calculated by the draftsperson, not as a mean of the lot distances on each section side line but as a distance weighted by the variance of the width of lot 4 from the normal width of 20 chains.
The distance between lots 2 and 3 is \((49.09 + 48.49) \div 4 = 24.395\).

Another solution would be to work from the east section line subtracting distances from acreages.
The distance between lots 1 and 2 is \(47.90 - 23.80 = 24.10\).
The distance between lots 2 and 3 is \(48.49 - 24.10 = 24.39\).

The solution is not the average of the parenthetical distances on opposite section lines, which would be \((23.80 + 24.80) \div 2 = 24.30\).

The distance between lots 5 and 6 is \((14.67 + 14.65) \div 4 = 7.33\), which happens to be the mean of the lot distances along the section lines, 7.31 and 7.35.

Another way to the solution is to work from the south section line subtracting distances from acreages.
The distance between lots 6 and 7 is \(14.69 - 7.35 = 7.34\).
The distance between lots 5 and 6 is \(14.67 - 7.34 = 7.33\).

The GLO Manual of Surveying Instructions of 1894 detailed a method for calculating lot distances for a section 6. A term \(q\) was assigned to the difference of the length of the east line of lot 1 and the length of the west line of lot 3. If \(q\) is determined, then add \(2/3\) of \(q\) to the east boundary of lot 1 to get the distance of the line between lots 2 and 3. Add or subtract, as appropriate, the product of the difference between the lengths of the lots along the section line times the ratio of the sum of the widths of lots 1 through 3 divided by the total width of the section.

Let: \(L(1\text{ east})\) be the length of the east line of lot 1.
\(L(4\text{ west})\) be the length of the west line of lot 4.
\(W(1-4)\) be the widths of lots 1 through 4.
\(W(1-3)\) be the widths of lots 1 through 3.
\(L(2\text{ west})\) or \(L(3\text{ east})\) be the length of the line between lots 2 and 3.

Then:
\[
q = [ L(1\text{ east}) - L(4\text{ west}) ] * [ W(1-3) \div W(1-4) ]
\]

\[
L(2\text{ west}) \text{ or } L(3\text{ east}) = L(1\text{ east}) \pm 2/3 \times q
\]

In the case of section 6 above \(q = (24.80 - 23.80) \times (60 \div 67.31) = 0.891\).
The distance between lots 2 and 3 = \(23.80 + 2/3 \times q = 24.39\), which is the answer derived from lot acreages.
The 1894 Manual method was not used for early GLO plats. Acreages should always be used to determine parenthetical distances. Section 6 below is an example.

The distance between lots 2 and 3 = (42.15 + 41.29) ÷ 4 = 20.86, which happens to be the mean of the lot side distances along the section lines, 21.72 and 20.00. The methods of the 1894 Manual were not used to plat this section.
Closing Sections with Closing Corners

For a discussion of establishing the true closing corner, of proportioning lost and subdivision corners along the closing line, and of establishing the quarter corner that was not established by the GLO survey refer to the single proportion chapter above in the discussion of the restoration of lost corners. For an example of a subdivision of a closing section against an irregular boundary see Appendix F.

Once the true closing corners are established, use the closing section subdivision procedures to complete the section subdivision.

Alternative section subdivision procedures have been used at times in the past. It is important to know about those procedures so that they can be recognized and respected in older surveys. An exaggerated situation with the closing corners far north of the standard parallel illustrates the differences in methodologies.

The figure below shows the Manual procedure. The north quarter-section corner is established by a cardinal offset from a point proportioned between the original closing corners. The sixteenth-section corners along the section line are proportioned using the original closing corners as control. The center-north sixteenth-section corner is proportioned using the true north quarter-section corner along the standard parallel as control. 

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25 Manual section 7-46. The quarter-section corners for sections on the side to which the closing corners refer were often not established in older surveys. The correct positions are protracted on the plat of these sections. When a new monument is to be established at the protracted quarter-section position, the original position of the section closing corners will control in the establishment. The proportionate measurement position between the original positions of the section closing corners will be moved in a cardinal direction ... to the true point on the line. This true point for the quarter-section corner will control the location of the legal subdivisions for the section on the side to which the closing corners refer.
A variation on the above manual procedure has been employed at times in the past. The north quarter-section corner was established by a cardinal offset from a position proportioned between the true closing corners on the standard parallel. Normally this would produce a position little different from the 2009 Manual procedure.

The figure below shows an alternative procedure that was sometimes used in the past. The center-north sixteenth-section corner was proportioned using the temporary quarter-section corner proportioned between the original closing corners. The true north quarter-section corner was established at an intersection of the standard parallel with the line connecting the temporary corner with the south quarter-section corner. This method can result in a significantly different location for the center-north sixteenth-section corner.
An example of such a closing section is the section 4 from the chapter on single proportion. The north quarter-section corner, not set by the GLO, and the north sixteenth-section corner on the west section line were calculated.

Section 4, T34N, R38E, W.M.

From the GLO plat the parenthetical distance between lots 2 and 3 is calculated:

\[(38.23 + 38.37) \div 4 = 19.15\]
\[(19.00 + 19.30) \div 2 = 19.15\]
The center-north sixteenth-section corner is proportioned using the calculated north quarter-section corner on the township line for control.

Total distance center of section to north quarter corner = 2581.05
Distance center of section to north sixteenth corner = 2581.05 * (20.00 ÷ 39.15) = 1318.54
Distance north sixteenth to north quarter corner = 2581.05 * (19.15 ÷ 39.15 ) = 1262.51
Appendices

Appendix A: Double Proportion

The 2009 Manual, page 167, presents a double proportion calculation method that can be employed only if the basis of meridian is true.

![Figure 7-2. Double proportionate measurement and cardinal equivalents.](image)

Figures 7-2 illustrates the calculations but leaves out some of the details.

The calculations in the leftmost of the three diagrams, the record between interdependent original corners, is self-explanatory, as are the calculations in the center diagram, the retracement between the temporary point and the found original controlling corners. The true line solution and the calculations at the bottom of the figure merit some explanation.

At the bottom of figure 7-2 the latitude and departure proportion calculations use the letters V and W twice. The first V is the latitude from B to X (the double proportioned position), the first W is the latitude of X to A. The second V is the departure from D to X, and the second W is the departure from X to C. These four numbers are shown in the box accompanying the true line solution in the figure. The figure does not have an explanation of how to calculate the departure of B to X, the departure of X to A, the latitude of D to X, and the latitude of X to C.
The following calculations assume coordinates of N=100 and E=100 at controlling corner B. In the calculations north latitude are plus and south latitudes are minus, east departures are plus and west departures are minus.

Coordinates of A, B, C, D, and T:
B:  N = 100;  E = 100
A:  N = (100 + 120.493) = 220.493;  E = (100+0.058-7.032) = 93.026
C:  N = (100+40.120+0.290) = 140.410;  E = (100+0.058-39.849) = 60.209
D:  N = (100+40.120-0.130) = 139.990;  E = (100+0.058+40.720) = 140.778
T:  N = (100+40.120) = 140.120;  E = (100+0.058) = 100.058

Calculate coordinates of X:

X:  N = [100(the northing of B) + 40.420(the proportioned latitude of B to X)] = 140.420
   E = [60.209(the easting of C) + 40.275(the proportioned departure of C to X)] = 100.484

Departure of B to X = 100.484(easting of X) - 100(easting of B) = +0.484
Departure of X to A = 93.026(easting of A) - 100.484(easting of X) = -7.458

Latitude of D to X = 140.420(northing of X) - 139.990(northing of D) = +0.430
Latitude of X to C = 140.410(northing of C) - 140.420(northing of X) = -0.010

Corner move:
Latitude from T to X = 140.420(northing of X)-140.120(northing of T) = +0.300
Departure from T to X = 100.484(easting of X)-100.058(easting of T) = +0.426

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Appendix B: Three-Point Control

T29N, R2W, W.M.

The coordinate system for the survey\textsuperscript{26} was Washington Coordinate System, North Zone, NAD 1927.

\textsuperscript{26} See Book 8 of Surveys, Page 148, records of Jefferson County, Washington, recorded 20 May 1987.
The coordinates of the controlling corners:
Southwest corner of section 35
337 355018.6470 N 47 57 16.09267 N
1491453.4560 E 122 54 32.10787 W
Convergence -01 32 43.13628  Scale Factor 0.999946308
Northwest corner of section 26
3252 365623.81400 N 47 59 00.91475 N
1492192.04300 E 122 54 25.45892 W
Convergence -01 32 38.18601  Scale Factor 0.999944993
Northeast corner of section 35
83 360213.74000 N 47 58 08.89751 N
1497318.74600 E 122 53 08.00494 W
Convergence -01 31 40.51994  Scale Factor 0.999945613

The north and south proportion:
This is a midpoint because the GLO measure is 80 chains for each mile.
337 355018.6470 1491453.4560
N 3°59'02"E 5315.427
x 360321.2305 1491822.7495 <- temporary point north and south
N 3°59'02"E 5315.427
3252 365623.8140 1492192.0430

Single point control from NE corner of Section 35:
GLO is N89°40'W, 80.18 chains. The cardinal equivalent is sin(89°40') * 80.18 = 80.1786 chs.
= 5291.790 feet. We are close to sea level so apply convergence of –1°31'41" and scale factor
of 0.9999456.
83 360213.74000 1497318.7460 N88°28'19"W 5291.503
y 360354.846 1492029.125 <- temporary point east and west

Now compute convergence and do cardinal intersection.
y 360354.846 N 1492029.125 E
Convergence -01 32 38.41461  Scale Factor 0.999945614
x 360321.231 1491822.749  S88°27'22"E 209.095
z 360315.697 1492028.070  N 1°32'38"E 39.163
y 360354.846 1492029.125

Inverse to controlling corners.
From z 360315.6968 1492028.0636:
S88°53’46"E, 5291.665 to 83: 360213.74000, 1497318.7460
S 6°11’28"W, 5328.125 to 337: 355018.64700, 1491453.4560
N 1°46’10"E, 5310.649 to 3252: 365623.81400, 1492192.043

The calculations shown are what were actually used in the survey. A more correct solution would have been to
used the convergence at the midpoint of the line from the NE corner of section 35 to the temporary point Y. That
would result in a “line of sight” with the same bearing as the “straight line” connecting the points. The difference
between the two methods for the final answer is in this case negligible because temporary point Y controls only the
departure (longitude) of the three-point control solution.

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Appendix C: Single Proportion

Quotations from the 2009 Manual relevant to single proportion.

Standard Parallel:
7-18. Restorations of lost corners of a standard parallel are controlled by the regular standard corners. These include the standard township, section, quarter-section, and sixteenth-section corners and meander corners. Also included are closing corners that were originally established by measurement along the standard line as points from which to start a survey and other corners that have been established by measurement in a retracement or dependent resurvey along the standard line.

Base Lines:
7-19. Corners on base lines are regarded the same as those on standard parallels.

Township Lines:
7-20. All lost section and quarter-section corners on the township boundary lines will be restored by single proportionate measurement... The control for either restoration should not extend beyond the township corner.

Double Corners:
7-21. Two sets of corners have been established on many township lines and on some section lines. Each set applies only to section on its respective side of the line. Which corners control the restoration of a lost corner will depend on how the line was surveyed.

Senior-Senior Corners:
7-22. Where both sets of corners have been established by measurement along the line in a single survey, each corner controls equally for both measurement and alinement. All corners are corners of maximum control.

Junior-Senior Corners:
7-23. This situation exists where one set of corners was established for one side of the line, and a second set of corners was established for the other side of the same line in the course of a later survey or retracement. The line is regarded as having been fixed in position by the senior survey and subsequent dependent resurveys or retracemenets. If both sets of corners are recovered, a junior survey, if it was established in the course of an obvious careful resurvey or retracement, reporting the most recent measurement of the line, will be used for alinement of the line and control in restoring a lost senior corner of the line.
7-26. In some older surveys, the policy was to establish junior corners without a careful retracement of the senior line. In those cases, a recovered junior corner not actually located on the line that was intended should not control the line for measurement or alinement. The new junior corner will be positioned in a cardinal direction ... from the original junior corner onto the line intended.
7-28. When a junior corner is recovered off the senior line and the new monument is established at the true corner point, the original position will control in the proportionate restoration of lost corners dependent upon the junior corner. The positioning of sixteenth-section corner(s) or lot
corner(s) on the junior line, will be based on the measurement to the original position of the junior corner.

Quarter-Section Corners:
7-35. All lost quarter-section corners on the section boundaries within the township will be restored by single proportionate measurement between the adjoining section corners, after the section corners have been identified or restored. In those cases where connections from the lost quarter-section corner to other regular corners of the line nearer than the section corners have been previously noted, these will ordinarily assume control in the restoration.

Meander Corners:
7-37. Lost meander corners, originally established on a line projected across the meanderable body of water, will usually be relocated by single proportionate measurement.

Closing Corners:
7-44. A lost closing corner on a standard parallel or other controlling boundary will be reestablished on the true line that is closed upon by single proportionate measurement between the nearest regular corners to the right and left of the lost corner.
Appendix D: Standard Parallel

Three different calculations to single proportion corners along a standard parallel. The GLO record was exactly 80 chains for every mile and 40 chains for every half-mile.

T10N, R39E, W.M.

Solution using the BLM program WinCMM

Here is the WinCMM geographic solution. Notice that every bearing is the same and represents a straight line that follows the latitudinal curve.

20034
(forward) S. 89°52'16" E. Dist. at mean elevation: 2000.00 ft.
(mean) S. 89°52'09" E. 1323.4901 ft. (20.053 ch)
(reverse) N. 89°52'02" W. Lat: -3.0241 Dep: 1323.4866
440100 = (894)
(forward) S. 89°52'22" E. Dist. at mean elevation: 2000.00 ft.
(mean) S. 89°52'09" E. 2646.9792 ft. (40.106 ch)
(reverse) N. 89°51'55" W. Lat: -6.0464 Dep: 2646.9723
500100 = (895)
(forward) S. 89°52'22" E. Dist. at mean elevation: 2000.00 ft.
(mean) S. 89°52'09" E. 2646.9809 ft. (40.106 ch)
(reverse) N. 89°51'55" W. Lat: -6.0476 Dep: 2646.9740
540100 = (896)
(forward) S. 89°52'22" E. Dist. at mean elevation: 2000.00 ft.
(mean) S. 89°52'09" E. 2646.9817 ft. (40.106 ch)
(reverse) N. 89°51'55" W. Lat: -6.0478 Dep: 2646.9757
600100 = (897)
WinCMM can convert the geographic solution into state plane inverses. Notice that here each bearing is different and represents a *line of sight* grid bearing between each of the corners of the line.

N. 88°15'25" E. 1323.2673 ft. (20.050 ch)  
Lat: 40.2490 Dep: 1322.6550  
440100 = (894)

N. 88°15'05" E. 2646.5336 ft. (40.099 ch)  
Lat: 80.7630 Dep: 2645.3010  
500100 = (895)

N. 88°14'37" E. 2646.5353 ft. (40.099 ch)  
Lat: 81.1130 Dep: 2645.2920  
540100 = (896)

N. 88°14'10" E. 2646.5361 ft. (40.099 ch)  
Lat: 81.4640 Dep: 2645.2820  
600100 = (897)

N. 88°13'42" E. 2646.5369 ft. (40.099 ch)  
Lat: 81.8160 Dep: 2645.2720

List Plane Coordinates:

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<tr>
<th>Point ID</th>
<th>Northing (ft.)</th>
<th>Easting (ft.)</th>
<th>Elevation (ft.)</th>
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Calculation using the Elgin, Knowles & Senne formula.

Formula for the offset to the latitudinal curve from POB Tech tips by Elgin, Knowles & Senne  
Offset = (0.6668) (LW) (LE) (Tan Latitude)

Where:
- Offset is in feet from the single proportioned point on a straight line, e.g. chord or secant of the latitudinal curve. (Note the BLM calls this a *line of sight*.)
- 0.6668 is a constant derived to make this equation simpler than it might be.
- LW is the distance west, in miles, from the single proportioned point to the known point.
- LE is the distance east, in miles, from the single proportioned point to the known point.
Single Proportion on a *Line of Sight*

Z14  361356.955  2291724.514  N88°14'30"E  1323.268
Z15T 361397.556  2293047.159  N88°14'30"E  2646.535
Z17T 361478.757  2295692.448  N88°14'30"E  2646.535
Z19T 361559.958  2298337.737  N88°14'30"E  2646.536
Z21T 361641.159  2300983.026  N88°14'30"E  2646.536
Z23  361722.360  2303628.316

Calculate grid direction and distance of offsets to the Latitudinal Curve. The offset in each case is in a cardinal direction.

Z15T  361397.556  2293047.159
Convergence: +1°52'33"  Direction: S1°52'33"E

(0.6668) * (1323.268 ÷ 5280) * (2646.535 * 4 ÷ 5280) * tan(46°17'42") = 0.351
Z15  361397.205  2293047.170

Z17T  361478.757  2295692.448
Convergence: +1°53'00"  Direction: S1°53'00"E

(0.6668) * ((1323.268 + 2646.535) / 5280) * (2646.535 ÷ 5280) * tan(46°17'42") = 0.787
Z17  361477.970  2295692.474

Z19T  361559.958  2298337.737
Convergence: +1°53'28"  Direction: S1°53'28"E

(0.6668) * ((1323.268 + 2646.535 * 2) ÷ 5280) * (2646.535 * 2 ÷ 5280) * tan(46°17'42") = 0.876
Z19  361559.082  2298337.766

Z21T  361641.159  2300983.026
Convergence: +1°53'55"  Direction: S1°53'55"E

(0.6668) * ((1323.268 + 2646.535 * 3) ÷ 5280) * (2646.535 ÷ 5280) * tan(46°17'42") = 0.614
Z21  361640.545  2300983.047

Inverse between final coordinates
Z14  361356.955  2291724.514  N88°15'25"E  1323.269
Z15  361397.205  2293047.170  N88°15'04"E  2646.536
Z17  361477.970  2295692.474  N88°14'37"E  2646.535
Z19  361559.082  2298337.766  N88°14'10"E  2646.535
Z21  361640.545  2300983.047  N88°13'43"E  2646.534
Z23  361722.360  2303628.316
Solution using Geographic Coordinates

Proportioning geographic coordinates puts the corners on a straight line, a rhumb line, following the latitudinal curve.

Z14
Latitude: 46° 17’ 42.31470”
Longitude: 117° 55’ 22.34649”

Z23
Latitude: 46° 17’ 42.04609”
Longitude: 117° 52’ 32.73184”

Differences in seconds:
Latitude 0.26861
Longitude 169.61465

The total distance is 180 chains.
The proportions are 1/9 the difference for 20 chains and 2/9 difference for 40 chains.

The resulting single proportioned coordinates
Z15
Latitude: 46 17 42.28485  Longitude: 117 55 03.50042
Northing/Y: 361397.204  Easting/X: 2293047.169

Z17
Latitude: 46 17 42.22516  Longitude: 117 54 25.80828
Northing/Y: 361477.966  Easting/X: 2295692.470

Z19
Latitude: 46 17 42.16547  Longitude: 117 53 48.11614
Northing/Y: 361559.080  Easting/X: 2298337.762

Z21
Latitude: 46 17 42.10578  Longitude: 117 53 10.42400
Northing/Y: 361640.544  Easting/X: 2300983.043
Appendix E: Irregular Boundary on an East and West Line

An irregular boundary adjustment of an east and west line requires that the retracement be done on the true meridian and that adjustment be made to the latitudinal curve. This is most easily accomplished using the BLM program CMM (Cadastral Survey Measurement Management). The GLO survey below shows a retracement of senior corners on the township line. A recent survey found many corners to be lost but found the southwest corner of section 31 and the south quarter corner of section 32. The intervening senior corners, the south quarter corner of section 31 and the section corner of sections 31 and 32, needed to be restored by an irregular boundary adjustment.

The GLO record for the mile and a half are:
N89°W, 40.09 chains
N89°08’W, 40.05 chains
N88°56’W, 39.28 chains

28 An alternative approximate method would be to rotate and scale each record course to state plane grid based on the convergence angle in the middle of the course and afterwards apply a compass rule adjustment.
The figures below show the two found corners and the proportioned corners in between. The basis of bearings is Washington State Plane, NAD 83 (1991), North Zone, US Survey Feet.

CMM Solution:

State Plane coordinates of the found corners:
297  1109155.0200   260411.5800  south quarter corner of section 32
598  1117038.8500   260147.3900  southwest corner of section 21

The CMM calculation will be done using geodetic coordinates and a mean elevation of 2840 feet:
297  47°41'36.56772"  122°59'26.97357"  2840.0000
     0.9999695309  1°36'23"              
598  47°41'36.12624"  122°57'31.66543"  2840.0000
     0.9999695466  1°34'57"

Enter the GLO record into CMM:
598
(forward)  N. 88°59'46" W.  Dist. at mean elevation: 2840.00 ft.
(mean)     N. 89°00'00" W.  2645.9400 ft.  (40.09 ch) (2645.94 ft)
(reverse)  S. 89°00'14" E.  Lat: 46.1780 Dep: -2645.5370
1
(forward)  N. 89°07'46" W.  Dist. at mean elevation: 2840.00 ft.
(mean)     N. 89°08'00" W.  2643.3000 ft.  (40.05 ch) (2643.30 ft)
(reverse)  S. 89°08'14" E.  Lat: 39.9815 Dep: -2642.9976
2
(forward)  N. 88°55'46" W.  Dist. at mean elevation: 2840.00 ft.
(mean)     N. 88°56'00" W.  2592.4800 ft.  (39.28 ch) (2592.48 ft)
(reverse)  S. 88°56'14" E.  Lat: 48.2610 Dep: -2592.0308
297
CMM reports the misclosure and adjusts the geodetic courses:

Report of Record Traverse Misclosure

Mean Geodetic Bearing to Closing Station:  S. 5°38'26" W.
Ground Distance to Closing Station:  90.1173 ft. at 2840.00 ft. Elev.
   Error in Latitude: 0°00'00.8849"
   Error in Longitude: 0°00'00.1295"
Error in State Plane Northing:  89.3826 ft.
   Error in State Plane Easting:  11.3657 ft.

Adjusted Courses

<table>
<thead>
<tr>
<th>Station</th>
<th>Bearing</th>
<th>Distance</th>
<th>Elev.</th>
<th>Lat.</th>
<th>Dep.</th>
</tr>
</thead>
<tbody>
<tr>
<td>598</td>
<td>47°41'36.12624&quot; 122°57'31.66543&quot;</td>
<td>90.1173 ft. at 2840.00 ft. Elev.</td>
<td>16.0724</td>
<td>-2648.5208</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>47°41'36.28484&quot; 122°58'10.37486&quot;</td>
<td>2648.5696 ft. (40.13 ch) (2648.57 ft)</td>
<td>9.9051</td>
<td>-2645.9746</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>47°41'36.38258&quot; 122°58'49.04711&quot;</td>
<td>2645.9931 ft. (40.09 ch) (2645.99 ft)</td>
<td>18.7621</td>
<td>-2594.9458</td>
<td></td>
</tr>
</tbody>
</table>

State Plane Inverses

<table>
<thead>
<tr>
<th>Station</th>
<th>Northing</th>
<th>Easting</th>
<th>Elev.</th>
<th>Lat.</th>
<th>Dep.</th>
</tr>
</thead>
<tbody>
<tr>
<td>598</td>
<td>260147.3900</td>
<td>1117038.8500</td>
<td>89.3764</td>
<td>-2646.6204</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>260236.7664</td>
<td>1114392.2296</td>
<td>83.5114</td>
<td>-2644.2346</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>260320.2778</td>
<td>1111747.9950</td>
<td>91.3022</td>
<td>-2592.9750</td>
<td></td>
</tr>
</tbody>
</table>

93
Appendix F: Irregular Closing Section

To subdivide the following closing section against an irregular boundary the challenge is to find the parenthetical distance along the east-west centerline. The methods used for closing sections against a *straight* boundary do not work. The calculations below produce acreages that fit the GLO plat. The methodology was found among some old GLO calculation sheets by the Oregon State Office of the BLM in Portland. Another method that could be tried is to run out the GLO bearings and distances around the section, compass adjust (maybe just the irregular boundary), and then subdivide the GLO section. If the resulting acreages match the plat, then the parenthetical distance between lots 2 and 3, would be evident.

GLO Plat of Section 13, T6N, R3E, W.M.

![GLO Plat of Section 13, T6N, R3E, W.M.]

Calculate Parenthetical Distances, Section 13, T6N, R3E, W.M.

Adjust out the misclosure in easting.

Tangent of bearing times northing = departure

<table>
<thead>
<tr>
<th>Bearing</th>
<th>Tangent</th>
<th>Northing</th>
<th>Departure</th>
</tr>
</thead>
<tbody>
<tr>
<td>-56'</td>
<td>-0.016291181</td>
<td>1.13</td>
<td>-0.018409034</td>
</tr>
<tr>
<td>-38'</td>
<td>-0.011054202</td>
<td>18.87</td>
<td>-0.208592795</td>
</tr>
<tr>
<td>-38'</td>
<td>-0.011054202</td>
<td>20.00</td>
<td>-0.221084043</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
\tan (-38') \times 0.87 &= -0.011054202 \times 0.87 = -0.009617156 \\
\tan (+1°12') \times 19.13 &= +0.020947014 \times 19.13 = +0.400716376 \\
\tan (+1°12') \times 20.00 &= +0.020947014 \times 20.00 = +0.418940278 \\
\text{Total} &= +0.361953627
\end{align*}
\]

<table>
<thead>
<tr>
<th>South Boundary of Lot 4</th>
<th>Computed total departures</th>
<th>Computed North Boundary of Lot 1</th>
<th>Actual North Boundary of Lot 1</th>
<th>Misclosure in easting</th>
</tr>
</thead>
<tbody>
<tr>
<td>= 17.73</td>
<td>= +0.36195363</td>
<td>= 18.09195363</td>
<td>= 18.40</td>
<td>= +0.308046370</td>
</tr>
</tbody>
</table>

Misclosure / Northing = Tangent of Angle of Misclosure in easting

\[+0.308046370 \div 80.00 = 0.003850580\]

### Adjust Course Tangents.

<table>
<thead>
<tr>
<th>Azimuth</th>
<th>Orig. Tan</th>
<th>Adjustment</th>
<th>Adjusted Tangent</th>
</tr>
</thead>
<tbody>
<tr>
<td>-56'</td>
<td>-0.016291181</td>
<td>+0.003850580</td>
<td>-0.012440601</td>
</tr>
<tr>
<td>-38'</td>
<td>-0.011054202</td>
<td>+0.003850580</td>
<td>-0.007203623</td>
</tr>
<tr>
<td>+1°12'</td>
<td>+0.020947014</td>
<td>+0.003850580</td>
<td>+0.024797594</td>
</tr>
</tbody>
</table>

Adjusted Tangent \times \text{Distance} = \text{Adjusted Departure}

\[-0.012440601 \times 1.13 = -0.014057879\]
\[-0.007203623 \times 18.87 = -0.135932359\]
\[-0.007203623 \times 20.00 = -0.144072453\]
\[-0.007203623 \times 0.87 = -0.006267152\]
\[+0.024797594 \times 19.13 = +0.474377962\]
\[+0.024797594 \times 20.00 = +0.495951868\]

Compute parentheticals south to north.

-0.014 = 17.716 at angle point
-0.136 = 17.580 between lots 3 and 4
-0.144 = 17.436 between lots 2 and 3 \(\text{-- this is the one we need}\)
-0.006 = 17.430 at angle point
+0.474 = 17.904 between lots 1 and 2
+0.496 = 18.400 north boundary of lot 1, check

Compute lot acreages south to north.

Lot 4 (1st part) = \(((17.73 + 17.716) / 2) \times 1.13) / 10 = 2.003
Lot 4 (2nd part) = \(((17.716 + 17.580) / 2) \times 18.87) / 10 = 33.302
Lot 4 (total) = 2.003 + 33.302 = 35.305 (record = 35.30)
Lot 3 = 17.580 + 17.436 = 35.016 (record = 35.02)
Lot 2 (1st part) = \(((17.436 + 17.430) / 2) \times 0.87) / 10 = 1.517
Lot 2 (2nd part) = \(((17.430 + 17.904) / 2) \times 19.13) / 10 = 33.797
Lot 2 (total) = 1.517 + 33.797 = 35.314 (record = 35.31)
Lot 1 = 17.904 + 18.400 = 36.304 (record = 36.30)